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MARKET FRICTIONS, ARBITRAGE, AND THE CAPITALIZATION OF AMENITIES

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ABSTRACT

The price-amenity arbitrage is a cornerstone of spatial economics, as the response of land and house prices to shifts in the quality of local amenities and public goods is typically used to reveal households' willingness to pay for amenities. With informational, time, and cash constraints, households' ability to arbitrage across locations with different amenities (demographics, crime, education, housing) depends on their ability to compare locations and to finance the swap of houses. Arbitrageurs with deep pockets and better search and matching technology can take advantage of price dispersions and unexploited trade opportunities. We develop a disaggregated search and matching model of the housing market with endogenously bargained prices, identified on transaction-level data from the universe of deeds for 6,400+ neighborhoods of the Chicago metropolitan area, matched with school-level test scores and geocoded criminal offenses. Price-amenity gradients reflect preferences and the capitalization of trading opportunities, which are arbitraged away in the frictionless limit. Thus the time-variation in hedonic pricing coefficients partly reflects the time variation in search and credit frictions. Our model is able to explain that, between the peak of the housing boom and its trough, the sign of the price-amenity gradient flipped, due to the decline in trading opportunities in lower-amenity neighborhoods and due to the lower capitalization of trading opportunities in house prices.

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1 Introduction

The capitalization of amenities into land and house prices changes over time. The relative prices of neighborhoods fluctuate substantially more than what changes in neighborhood amenities would imply, even when accounting for the expected flow value of future neighborhood amenities. Rationalizing the change in such capitalization with shifts in the distribution of preferences requires substantial shifts in buyers' and sellers' preferences for amenities. In the city of Chicago between 2007, right before the mortgage bust, and 2010, during the trough of the mortgage crisis, prices in neighborhoods with low school test scores and high crime rates collapse dramatically compared to neighborhoods with better schools and lower crime, after controlling for a large array of observables and unobservable characteristics. This paper proposes and estimates a disaggregated dynamic model of housing choices with endogenous prices, search and credit frictions that can explain such changes in the capitalization of amenities into individual house prices. The model explains housing decisions as resulting from preferences but also from trading opportunities, including the opportunity to "trade-up" in the future. While the distribution of preferences is stable over time, trading opportunities change drastically, reflecting time-variation in credit and matching frictions, and especially so time-variation in mortgage indebtedness, interest payments, and approval rates. We find that such variations explain a significant part of the variation in the time-varying capitalization of amenities. The model is estimated for Chicago over the period 2005-2013 taking advantage of the drastic changes in credit conditions over the period.

A key novel insight of our dynamic approach is to show how neighborhood dynamics depend on a simple *index of trading opportunities,* which is defined as the difference between the lifetime expected value of living in a neighborhood − which includes the flow value of amenities and the option to move out of it at some point − and the lifetime expected value of owning, maintaining, and receiving potential rental payments for a vacant home in the same neighborhood; the latter value is also the reservation value, i.e. the lowest price at which its owner is ready to sell the unit. The index of trading opportunities determines the relative attractiveness of neighborhoods in the city. Homeowners would like to leave neighborhoods for which the index of trading opportunities is low − that is the amenity value of their home is low but their market value is high − and to move into a neighborhood for which the opposite is true. The difference of index of trading opportunities between two neighborhoods is exactly equal to the joint surplus of transactions between a buyer moving into a unit and a seller selling a vacant home.

The key role played by trading opportunities establishes a new link between two key areas of economics: the literature on price-amenity trade-offs in housing choices, and the literature on the limits to arbitrage in asset markets. We show that in the absence of search and credit frictions, deep-pocketed arbitrageurs could fully exploit existing arbitrage opportunities across neighborhoods. They would do so by buying a vacant home in neighborhoods where the index of trading opportunities is low, and sell it to buyers in neighborhoods in which the index of trading opportunities is high; by this process, the index of trading opportunities is equalized across neighborhoods. Therefore the persistence of differences in trading opportunities across neighborhoods reveals the limits to arbitrage due to search and credit frictions, which may prevent households from "trading-up" their home for another for which the price-amenity trade-off is more favorable.

Individual households face search and mortgage credit frictions, and their forward-looking decisions to buy and sell houses are constrained by such frictions. When purchasing a new house, the homeowner vacates his current house and lists it on the market. When making offers, home buyers consider the value of enjoying the amenities of the new home, as well as the value of moving out and putting their existing house on the market. In doing so, moving in to a new unit depends on the comparison of trading opportunities across neighborhoods. Mortgage credit conditions are an important source of friction faced by households when making their location choices. Borrowers with a mortgage balance that exceed their value of their house face limited mobility options. Indebted homeowners are exposed to both interest rate risk and also risk to the value of their equity, endogenously pinned down by the market equilibrium. The ability to secure a mortgage for a new home depends crucially on the build-up of equity in the previous home as well as current lending standards.

This paper's dynamic general equilibrium model of housing choices incorporates search and credit conditions in housing choices. With constant preferences, increases in housing market liquidity and the relaxation of credit conditions leads to substantial shifts in the ability of households to arbitrage between locations, and thus to shifts in the price-amenity relationship. We show that evolution of the index of trading opportunities across neighborhoods, the demographics of inflows and outflows, and the dynamics of mortgage balances are jointly sufficient to fully identify households' preferences for amenities, the matching technology, credit frictions and households' time discount factor. In turn, such parameters fully characterize city dynamics, including listings, sales, transaction prices, and flows.

The central novel theoretical result of the paper is the exact derivation of the price-amenity gradient as the sum of three components: the current flow of utility of owning a house, the expected future utility flows of owning a house, and the trading opportunities-amenity gradient weighted by capitalization of trading opportunities into prices. Variation in credit market frictions affects the price-amenity gradient through both the dynamics of trading opportunities and the variation of its capitalization into house prices. A tightening of credit frictions implies that households will be less able to seize future trading-up opportunities, which, in turn, reduces their mobility and thus the sale of housing units to households from other neighborhoods. This causes a decline in the neighborhood's current index of trading opportunities. For a giving set of trading opportunities, the tightening of credit constraints also reduces the set of potential trades that generate a surplus for buyers and sellers. This corresponding reduction in sales reduces the capitalization of trading opportunities into prices. The combined variations in trading opportunities and its capitalization into prices lead to variations in the price-amenity gradient.

The model is identified using flows of buyers to each of the 6,400+ neighborhoods and the corresponding transaction prices. The difference with a Hotz & Miller (1993) structural estimation approach is that, in this model with endogenously Nash-bargained prices, the flows identify trading opportunities rather than values. Hence the 2005-2013 gross flows to each of the neighborhoods yield the trading opportunities across locations by demographic subgroup. The observation of prices disentangles the value of owner occupation from the value of vacant or rented out units. By projecting values on a flexible polynomial of amenities, we can identify the functional form tying values to amenities, and then estimate the expectation of future values for each neighborhood using numerical integration. Annual vacancies by neighborhood provide matching probabilities. Observed approval rates, current mortgage balances, and expected interest cost payments provide estimates of credit frictions. Finally, the regression of current values on current amenities and expectations, potentially affected by credit frictions, identifies the time discount factor. These steps fully identify preferences, the matching technology, credit frictions, and the time discount factor.

The model is estimated using the universe of transactions, the universe of mortgages, and mortgage applications for the Chicago metropolitan area between 2005 and 2013, a period that includes the end of the credit boom, the mortgage crisis, and the post-crisis recovery. While credit conditions and transaction prices experience large variations, the estimated model delivers stable estimates of households' preferences for amenities. Importantly, incorporating mortgage credit characteristics significantly impact the estimates of household preferences, and in particular their discount factor and preferences for same-race neighbors.

The empirical analysis shows how the model-estimated decomposition of the price-amenity gradient can account for its drastic steepening between 2007 (the peak of the housing boom) and 2010 (the trough of the mortgage crisis). Between 2007 and 2010, the neighborhoods at the bottom of the distribution of amenities, which are for a large part located in the West-Side and South-Side of Chicago, experience larger increases in mortgage balances, larger decreases in trading opportunities, and large declines in house prices compared to the rest of the MSA. In contrast, controlling for trading opportunities yields a very stable relationship between prices and amenities over the period 2005-2013. This yields the key empirical result of the paper: fluctuations in the price-amenity gradients are mostly explained by fluctuations in the distribution of trading opportunities, and their capitalization into prices, rather than by variations in current and future utility flows. Consistent with the model's predictions, the fall in trading opportunities and of their capitalizations into prices appear partly driven by the tightening of credit frictions, and especially so by the increase in the ratio of mortgage balances over current house prices, and the associated rise of "*underwater*" mortgages.

The paper's results have substantial implications for the measurement of households' preferences for public goods. Households' trade-off between house prices and amenities has underpinned the literature on the capitalization of amenities, as equilibrium prices are determined by households' willingness to pay for locations (Alonso et al. 1964, Rosen 1974, Roback 1982, Benabou 1996). The relationship between house prices and households' preferences for amenities is a key way to measure the dollar value of benefits of public good investment in education (Black 1999), the environment (Cragg & Kahn 1997), crime reduction (Linden & Rockoff 2008), and transportation infrastructure (Chen & Haynes 2015).

This paper contributes to an emerging literature that highlights the importance of dynamic considerations in the understanding of relationships between prices and amenities. Bishop & Murphy

(2011) shows that hedonic pricing estimates based on current amenities underestimate the preference for amenities when such amenities are mean-reverting. This paper endogenizes the 3-way neighborhood-level price negotiation between buyer, seller, and lender. Endogenous prices reflects both local market tension and time-varying credit frictions, which is key in obtaining precise and stable estimates of household preferences. Furthermore, understanding the dynamics of such frictions and their impact on trading opportunities explains a substantial share of time-variations in the price-amenity relationship.

This paper connects the literature on urban choices with the literature on the limits to arbitrage in assets markets. In finance, a series of seminal papers shows how financing constraints can lead to mispricing (Shleifer & Vishny 1997, Gromb & Vayanos 2002, Gromb & Vayanos 2015), as such constraints imply that long-run prices can differ from fundamentals. In housing markets, given the greater level of differentiation, segmentation in search, and the lower levels of liquidity, mispricing is also a likely salient feature and arbitrage opportunities are arguably pervasive. For instance, Glaeser & Gyourko (2007) provides evidence that the no-arbitrage rental-ownership relationship may not hold. Within the set of differentiated locations of the city, this paper provides a closed–form expression describing how search costs and credit frictions imply long-run differentials between prices and the value of amenities; professional arbitrageurs with access to better search technology and/or less liquidity constraints than individual households may take advantage of mispricing and narrow the distribution of prices.[1] This paper's structural estimates pinpoint the spatial location of such arbitrage opportunities.

The paper also contributes to the emerging literature that describes the importance of the within-city dispersion of house prices along the business cycle (Liu, Nowak & Rosenthal 2016). Following the reasoning of recent contributions (Bayer, Ferreira & McMillan 2007, Bayer, McMillan, Murphy & Timmins 2016), such within-city variations could be explained by describing how amenities are priced at different percentiles of the distribution of households' preferences in different parts of the business cycle – generating cyclicality in the pricing of amenities without the need to introduce market frictions. Other key contributions (Gabriel, Hearey, Kahn & Vaughn 2016) describe how preferences for amenities *themselves* vary across the business cycle, e.g. as the returns to education

[1] Combined with access to more sophisticated market information, companies such as *Opendoor* and *Zillow* have started engaging in directly buying and selling; and thus may resemble this paper's arbitrageurs. A short introduction to these new players is featured in Manjoo (2017).

can be higher during a boom. This paper suggests that the capitalization of amenities can vary even without preference heterogeneity and without temporal variation in households' willingness to pay for amenities due to changes in credit frictions affecting the ability of households to exploit trading opportunities.

The paper is structured as follows. Section 2 presents the theoretical framework: the model of the city with differentiated locations and endogenous prices; the existence of a steady-state; and the short- and medium-run transition dynamics to the long-run steady-state. Section 3 provides a closed-form expression of the price-amenity gradient as a function of preferences, trading opportunities, search and credit frictions. Section 4 identifies the city's structural parameters (preferences, matching technology, and time discount factors). Section 5 introduces the transaction-level data from the 14 counties of the Chicago metropolitan area. Section 6 presents the estimates of household preferences and empirically analyzes the role of credit frictions in the evolution of the price-amenity gradient. Section 7 concludes.

2 Theoretical Framework

2.1 The Model

The city is composed of J neighborhoods indexed by j. The j-th neighborhood has h_j housing units. The observable amenities of neighborhood j are noted $\mathbf{z}_j$, a vector of size K. The unobservable amenities are a J-vector of scalars ε_j with variance σ_ε^2. Stacked into a $J \times (K+1)$ matrix $\mathbf{Z}$, such matrix describes the distribution of amenities across the J neighborhoods. There is an infinite number of discrete time periods $t = 1, 2, \ldots$. The distribution of amenities in the next period conditional on the current matrix of amenities is $\mathbf{Z}'|\mathbf{Z}$.

There are $i = 1, 2, \ldots, N$ households in the city, each living in one housing unit. The total number of households in the city is less than the number of housing units, $N < H$. The number of households living in neighborhood j when the matrix of amenities is $\mathbf{Z}$ is noted $n(j, \mathbf{Z})$. The $H - N$ vacant housing units are owned by households of the city. We assume that $L = H - N < 2H$.

The value of living in neighborhood j when the distribution of amenities in the city is $\mathbf{Z}$ is noted $V_i^\ell(j, \mathbf{Z}) \in \mathbb{R}$. The value of owning a vacant unit in neighborhood j is noted $V_i^v(j, \mathbf{Z}) \in \mathbb{R}$.

Preferences for the K observable amenities are described by a vector $\boldsymbol{\xi}_i$, and unobservable amenities are a term ε_j^i so that the flow utility enjoyed in one period by a household living in j is $\mathbf{z}_j\boldsymbol{\xi}_i + \varepsilon_j^i$. The unobservables ε_j^i are i.i.d. and are normally distributed with cumulative distribution function $F(.)$, the c.d.f of the normal with variance σ_ε^2.

Without loss of generality, and for the sake of notational simplicity, the rest of the exposition proceeds without individual household indices i. Yet, the framework can accommodate heterogeneity in preferences for amenities and heterogeneity in frictions.

Each period, a household occupying a house in neighborhood j is matched to a listed vacant house in neighborhood j' with probability $\delta\lambda_{j'}$, with $\sum_{j'}\lambda_{j'} = 1$. When a household living in j is matched to a listed vacant house in neighborhood j', they bargain over the transaction price $\log p(j, j', \mathbf{Z})$. If the household in j buys a vacant house in j', she pays price $\log p(j, j', \mathbf{Z})$, moves to neighborhood j', and owns a vacant house in j that is listed. The seller simply obtains the price $\log p(j, j', \mathbf{Z})$ for the vacant unit and transfers the property right to the buyer. There can thus a mutually beneficial transaction if there exists a price $\log p(j, j', \mathbf{Z})$ such that:

$$V^v(j', \mathbf{Z}) \leq \log p(j, j', \mathbf{Z}) \leq V^\ell(j', \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^\ell(j, \mathbf{Z}), \tag{1}$$

in which case we write $\mathbf{1}(j, j', \mathbf{Z}) = 1$, and 0 otherwise. The Nash-bargaining power of the seller is noted γ so that, when a transaction occurs the price is:

$$\begin{aligned}\log p(j, j', \mathbf{Z}) = \gamma \cdot \Big[V^\ell(j', \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^\ell(j, \mathbf{Z})\Big] \\ + (1-\gamma)\cdot V^v(j', \mathbf{Z}).\end{aligned} \tag{2}$$

The value of living in neighborhood j is thus the sum of the flow utility and expectations, discounted by an intertemporal discount factor β:[2]

$$\begin{aligned}V^\ell(j, \mathbf{Z}) = \mathbf{z}_j\boldsymbol{\xi} + \beta\Big[&(1-\delta)\mathbb{E}V^\ell(j, \mathbf{Z}') \\ &+ \delta\sum_{j'=1}^{J}\lambda_{j'}\mathbb{E}\{(\mathbf{1}(j, j', \mathbf{Z}')(V^\ell(j', \mathbf{Z}') + V^v(j, \mathbf{Z}') - \log p(j, j', \mathbf{Z})) \\ &+ (1-\mathbf{1}(j, j', \mathbf{Z}'))V^\ell(j, \mathbf{Z}')\}\Big] + \varepsilon^l,\end{aligned} \tag{3}$$

[2]The symbol $\mathbb{E}$ denotes expectations of $t+1$ while the symbol E is the average over a distribution.

where ε^l is a normally distributed unobservable preference term. The owner of a vacant unit in j receives a flow $\mathbf{z}_j\zeta$ that reflects maintenance and the potential flow of rental payments. A vacant unit is matched to an owner in neighborhood j' with probability $\delta^v\mu_{j'}$. Thus the value of owning a vacant unit is:

$$V^v(j,\mathbf{Z}) = \mathbf{z}_j\zeta + \beta\Bigg[(1-\delta^v)\mathbb{E}V^v(j,\mathbf{Z}') \\ + \delta^v\sum_{j'=1}^{J}\mu_{j'}\mathbb{E}\{\mathbf{1}(j',j,\mathbf{Z}')\log p(j',j,\mathbf{Z}') + (1-\mathbf{1}(j',j,\mathbf{Z}'))V^v(j,\mathbf{Z}')\}\Bigg] + \varepsilon^v. \quad (4)$$

where ε^v is a normally distributed unobservable preference term. The change in the number of listings in j is noted $\Delta\ell(j,\mathbf{Z})$ and is a function of the number of inflows (vacant units being sold to households, leaving the listing), and outflows (occupied housing units becoming vacant and listed):

$$\Delta\ell(j,\mathbf{Z}) = E\left[n(j,\mathbf{Z})\delta\sum_{j'=1}^{J}\lambda(j',\mathbf{Z})\mathbf{1}(j,j',\mathbf{Z}) - \ell(j,\mathbf{Z})\delta^v\sum_{j'=1}^{J}\mu(j',\mathbf{Z})\mathbf{1}(j',j,\mathbf{Z})\right], \quad (5)$$

where $L = \sum_{j=1}^{J}\ell(j,\mathbf{Z})$ is the total number of listings. E denotes the average over all matches, whereas $\mathbb{E}$ is an intertemporal expectation. In this version of the model the probability of being matched to a vacant house in j' is simply proportional to the number of listed houses in j', and the probability of being matched to a buyer in j' is simply proportional to the number of households in j':

$$\lambda(j',\mathbf{Z}) = \frac{\ell(j',\mathbf{Z})}{L}, \qquad \mu(j',\mathbf{Z}) = \frac{n(j',\mathbf{Z})}{N}. \quad (6)$$

The framework can be extended to allow for a *segmented housing market*.

2.2 Steady-State City Dynamics

A key role is played by the difference $\nabla(j,\mathbf{Z}) = V^\ell(j,\mathbf{Z}) - V^v(j,\mathbf{Z})$ between the value of living in a neighborhood and the value of owning a vacant housing unit, called the *trading opportunities index*. Indeed, city dynamics can be fully described by only two sets of quantities: (i) the difference $\nabla(j,\mathbf{Z}) = V^\ell(j,\mathbf{Z}) - V^v(j,\mathbf{Z})$ between the value of living in a neighborhood and the value of owning a vacant unit, and (ii) the listings in neighborhood j, $\ell(j,\mathbf{Z})$. The index of trading opportunities $\nabla(j, \mathbf{Z})$ satisfies a simple Bellman equation:

$$\begin{aligned}\nabla(j,\mathbf{Z}) &= \mathbf{z}_j(\boldsymbol{\xi}-\boldsymbol{\zeta}) + \beta\mathbb{E}\nabla(j,\mathbf{Z}') \\ &\quad + \beta\mathbb{E}\Bigg[\sum_{j'=1}^{J}\big\{(1-\gamma)\delta\lambda(j',\mathbf{Z}')\boldsymbol{\Phi}(\nabla(j',\mathbf{Z}')-\nabla(j,\mathbf{Z}')) \\ &\qquad + \gamma\delta^v\mu(j',\mathbf{Z}')\boldsymbol{\Phi}(\nabla(j,\mathbf{Z}')-\nabla(j',\mathbf{Z}'))\big\}\Bigg] + \varepsilon, \end{aligned} \tag{7}$$

where $\boldsymbol{\Phi}(d) = F(\frac{d}{\sqrt{2}\sigma_\varepsilon})d + \sqrt{2}\sigma_\varepsilon f(\frac{d}{\sqrt{2}\sigma_\varepsilon})$, which captures the expected value of the unobservables conditional on a transaction. The unobservable is $\varepsilon = \varepsilon^l - \varepsilon^v$ and $\sigma_\varepsilon^2 = Var(\varepsilon)$. The first term of the equation corresponds to the difference between the current utility flows of living-in in the unit and that of owning a vacant home. The second term is the expected value of future trading opportunities. The last two terms correspond to the expected values attached with moving-up to a different housing unit, and that of selling a vacant housing unit, respectively. The dynamic of ∇ implies transaction probabilities, values V^l and V^v, and prices, which depend solely on such trading opportunities. For instance, a transaction occurs in j from j' with probability $F(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))$, i.e. is a function of whether such difference is higher in neighborhood j than in neighborhood j'.

This expression for the difference in values, together with population dynamics (5), defines city dynamics:

$$\begin{aligned}\Delta\ell(j,\mathbf{Z}) = \Bigg\{&\delta n(j,\mathbf{Z})\sum_{j'=1}^{J}\lambda(j',\mathbf{Z})F\left[\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z})\right] \\ &- \delta^v\ell(j,\mathbf{Z})\sum_{j'=1}^{J}\mu(j',\mathbf{Z})F\left[\nabla(j',\mathbf{Z})-\nabla(j,\mathbf{Z})\right]\Bigg\},\end{aligned} \tag{8}$$

and $\Delta n(j,\mathbf{Z}) + \Delta\ell(j,\mathbf{Z}) = 0$: every net population increase matches a net decline in listings. where the model's stock-flow structure ensures that the total household count $\sum_j n(j,\mathbf{Z}) = N$ at every point. The city is fully described by $\boldsymbol{\nabla}$ and $\Delta\boldsymbol{\ell}$ only.

The following proposition shows that a unique steady-state equilibrium of the city exists.

Proposition 1. (Existence and Uniqueness of a Steady-State City Equilibrium) *Given the structural parameters of household preferences $\boldsymbol{\xi}$, time discount factor β, the vector of housing units $\mathbf{h}$, an equilibrium of the city is a couple (∇^*, ℓ^*) made of a continuous and differentiable* *function* $\nabla^* : \{1, 2, \ldots, J\} \times \mathbb{R}^{JK} \to \mathbb{R}$ *and a continuous and differentiable function of listings* $\ell^* : \{1, 2, \ldots, J\} \times \mathbb{R}^{JK} \to \mathbb{R}$ *for each neighborhood and for each city-wide distribution of amenities* $\mathbf{Z}$ *that satisfy the dynamics (7) and for which the net change in listings is* $\Delta\ell(j, \mathbf{Z}) = 0$ *in (8). When* $\max\{\beta, \delta, \delta^v\} < 1$*, such an equilibrium exists and is unique. The value functions* $\mathbf{V}^\ell$ *and* $\mathbf{V}^v$*, the transaction probabilities* $\mathbf{F}$ *and prices* $\log \mathbf{p}$ *are implied by the couple* $(\nabla, \boldsymbol{\ell})$.

Proof. Presented in Appendix Section D.1. □

2.3 Introducing Credit Frictions

We introduce three key dimensions of credit frictions: approval rates for mortgage applications, mortgage balances, interest costs. This has two separate impacts on city dynamics: first, the lender may not approve mortgage applications thus lowering the volume of transactions. Second, the lender captures part of the transaction surplus in the form of interest payments. Third, a household who moves out pays his mortgage balance, reducing the surplus of a transaction and lowering the volume of transactions.

Approval Probability

The approval rate probability $\varphi(j, \mathbf{Z}; \mathbf{x})$ depends on neighborhood and household characteristics.

$$\varphi(j, \mathbf{Z}; \mathbf{x}) = \Lambda(-a \log \mathbf{p} + \mathbf{z}'c + \mathbf{x}'b + \mathbf{z}'d\mathbf{x}) \tag{9}$$

where a, c, b, d are constants. $\log \mathbf{p}$ is the endogenously negotiated price. Overall, a transaction occurs if it is mutually beneficial for the buyer and the seller *and* if the lender approves the mortgage. Thus a transaction occurs with a probability that is the product of the two probabilities.

Transaction Surplus

Prior to moving out of their unit, households pay their outstanding mortgage balance, noted $B(j, \mathbf{Z})$. When taking a loan, borrowers take into account the interest cost $\kappa(j', \mathbf{Z})$ on top of the purchase price. Both the balance payment and the anticipation of an interest payment affect the buyer's surplus. Indeed, the value of moving into j' is $V^\ell(j', \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^l(j, \mathbf{Z}) - B(j, \mathbf{Z}) - \kappa(j', \mathbf{Z})$.

The interest cost $\kappa(j', \mathbf{Z})$ is the present discounted value of interest payments on the annual mortgage balance. For a $T = 30$ year fixed rate mortgage this is simply:

$$\kappa(j', \mathbf{Z}) = \sum_{t=0}^{T} \beta^t \log [rB_t] \tag{10}$$

where B_t is the balance in year t, r is the mortgage's interest rate. For a 30-year fixed rate mortgage with payment m and loan-to-value LTV, then $T = 30$ and the balance at t is simply $B_t = m + (1 + r)^t(r \cdot LTV \cdot p - m)$.

Transaction probability and price

As in the model without friction, the transaction price splits the transaction surplus between the buyer and the seller, where the seller's bargaining power is noted γ. Here, the value of moving into a housing unit in neighborhood j' is affected by the balance and the interest cost payment:

$$\begin{aligned}\log p(j, j', \mathbf{Z}) = \gamma \cdot (V^\ell(j', \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^\ell(j, \mathbf{Z}) \overbrace{-B(j, \mathbf{Z}) - \kappa(j', \mathbf{Z})}^{Credit\, Frictions}) \\ + (1 - \gamma) \cdot V^v(j', \mathbf{Z})\end{aligned} \tag{11}$$

And the probability of a successful transaction turns into:

$$\pi(j, j', \mathbf{Z}) = \varphi(j', \mathbf{Z}) F(\nabla(j', \mathbf{Z}) - \kappa(j', \mathbf{Z}) - B(j, \mathbf{Z}) - \nabla(j, \mathbf{Z})) \tag{12}$$

where $\varphi(j', \mathbf{Z})$ is the probability of approval in neighborhood j'.

In an extension of the model we also introduce the possibility of foreclosures.

Steady-State City Dynamics with Credit Frictions

The evolution of the city with credit frictions is driven by three dynamics: (i) the dynamics of trading opportunities, described in equation (7); the difference here being that opportunities to trade-up are constrained by the approval rates, the mortgage balance, and the interest cost. (ii) the dynamics of listings. (iii) the dynamic of mortgage balances $B(j, \mathbf{Z})$. The first two are similar to the model without credit frictions and are presented in Appendix (A). The third one is specific to the city with credit frictions.

The dynamic of mortgage balances is as follows. Mortgage balances decline due to the flow of households who move out of the neighborhood and repay. Mortgage balances increase for new inward movers who take up new loans. Finally, mortgage balances decline when households repay their loans during the maturity of the mortgage.[3]

$$\begin{aligned}\Delta B(j,\mathbf{Z}) = & \overbrace{-n(j,\mathbf{Z})\delta\sum_{k=1}^{J}\lambda_k\pi_{jk}\cdot B(j,\mathbf{Z})}^{Households\,moving\,out} \\ & + n(j,\mathbf{Z})\left(1-\delta\sum_{k=1}^{J}\lambda_k\pi_{jk}\right)\cdot[rB(j,\mathbf{Z})-m(j,\mathbf{Z})] \\ & + \underbrace{(h_j-n(j,\mathbf{Z}))\cdot\delta^v\sum_{k=1}^{J}\mu_k\pi_{kj}\cdot LTV\cdot p(k,j,\mathbf{Z})}_{Households\,moving\,in} \end{aligned} \tag{13}$$

At the steady-state, the city experiences no net shift in mortgage balances, $\Delta B(j,\mathbf{Z})=0$.

Proposition 2. (Existence and Uniqueness of City Equilibrium with Frictions) *Given the structural parameters* $(\boldsymbol{\xi},\beta,\mathbf{h})$ *and the mortgage credit conditions* $(\boldsymbol{\varphi},r,T)$*, an equilibrium of the city is a triplet* $(\nabla,\ell,\mathbf{B})$ *made of a function* $\nabla:\{1,2,\ldots,J\}\times\mathbb{R}^{JK}\to\mathbb{R}$*, a function of listings* $\ell:\{1,2,\ldots,J\}\times\mathbb{R}^{JK}\to\mathbb{R}$*, and a function of mortgage balances* $B:\{1,2,\ldots,J\}\times\mathbb{R}^{JK}\to\mathbb{R}$ *for each neighborhood and for each city-wide distribution of amenities* $\mathbf{Z}$ *that satisfy the dynamics (7), for which the net change in listings is* $\Delta\ell(j,\mathbf{Z})=0$ *in (8), and the change in mortgage balances is* $\Delta B(j,\mathbf{Z})$*. Such an equilibrium exists and is unique in the space of continuous and differentiable functions. The value functions* $\mathbf{V}^{\ell}$ *and* $\mathbf{V}^{v}$*, the transaction probabilities* $\mathbf{F}$ *and prices* $\log\mathbf{p}$ *are implied by the couple* $(\nabla,\boldsymbol{\ell},\mathbf{B})$*.*

Proof. The proof is generically similar to the proof for the model without credit frictions (Proposition (1)) with the addition of the dynamics of balances $\mathbf{B}$. As in the prior proof, write $\mathbf{B}=T_B(\boldsymbol{\nabla},\boldsymbol{\ell},\mathbf{B})$ the dynamic of mortgage balances (13), and realize that T_∇ and T_ℓ are also a function of the vector of balances $\mathbf{B}$. The operator T_B is a contraction in the space $\mathbb{S}$ of triplets $(\boldsymbol{\nabla},\boldsymbol{\ell},\mathbf{B})$

[3] Keeping track of the mortgage balance of each household in each neighborhood would dramatically increase the dimensionality of the dynamics. We choose a parsimonious representation where $B(j,\mathbf{Z})$ measures the average mortgage balance in neighborhood j. When we introduce household heterogeneity by observable $\mathbf{x}$, this turns into $B_{\mathbf{x}}(j,\mathbf{Z})$.

and such triplets $(\boldsymbol{\nabla}, \boldsymbol{\ell}, \mathbf{B})$ belong to a bounded set for the norm $|||\cdot|||_\infty$ on $\mathbb{S}$ defined as the max of $(\boldsymbol{\nabla}, \boldsymbol{\ell}, \mathbf{B})$. The mapping $T = (T_\nabla, T_\ell, T_B)$ is a contraction as long as $\max\{\beta, \delta, \delta^v\} < 1$. It admits a unique fixed point in the Banach space $(\mathbb{S}, |||\cdot|||_\infty)$. □

3 The Price-Amenity Gradient at Equilibrium

3.1 Current Amenities vs. Trading-Up Opportunities

In a dynamic setting, holding other neighborhood amenities constant, a marginal increase in the quality $\mathbf{z}$ of current amenities in a specific neighborhood j leads to two separate impacts on transaction prices in that neighborhood: first, the value V^ℓ of living in the neighborhood increases; second, the value of selling the housing unit (once vacated) V^v changes. With Nash bargained prices according to 2, the impact of $d\mathbf{z}_j$ on the price of a transaction for a buyer coming from k buying in j is the sum of these two effects:

$$\frac{\partial \log p(k, j, \mathbf{Z})}{\partial \mathbf{z}_j} = \gamma \cdot \frac{\partial V^\ell(j, \mathbf{Z})}{\partial \mathbf{z}_j} + (1-\gamma) \cdot \frac{\partial V^v(j, \mathbf{Z})}{\partial \mathbf{z}_j} \tag{14}$$

In turn, the impact of a shift $d\mathbf{z}_j$ in amenities on the value V^v of living in the unit is an aggregation of the preference for amenities, the expectation of future amenities, and the opportunities to 'trade-up' to a better unit.

$$\begin{aligned} \frac{\partial V^\ell(j, \mathbf{Z})}{\partial \mathbf{z}_j} = \boldsymbol{\xi} + \quad & \overbrace{\beta \frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^\ell(j, \mathbf{Z}')|\mathbf{Z}\right]}^{\text{Expectations of future amenities}} \\ & \underbrace{- \beta\delta(1-\gamma) \sum_{j'=1}^{J} \lambda_{j'} \Phi'(\nabla(j', \mathbf{Z}) - \nabla(j, \mathbf{Z})) \frac{\partial \nabla(j, \mathbf{Z})}{\partial \mathbf{z}_j}}_{\text{Opportunities to trade up}} \end{aligned} \tag{15}$$

Thus a household with few opportunities to trade up (in the extreme, e.g. if $\delta = 0$, $\beta = 0$, or $\gamma = 1$) will see the impact of amenities only through the flow value of amenities and not through trading-up opportunities. The value of living in the unit depends negatively on the trading opportunities index $\nabla(j, \mathbf{Z})$ as a household living in a neighborhood with a more favorable $\nabla(j, \mathbf{Z})$ is less likely to move and will thus trade-up less often.

The impact of the amenity shift on the resale value V^v of the vacant unit in 14 depends on the preferences of renters $\boldsymbol{\zeta}$ and the trading opportunities index $\nabla(j, \mathbf{Z})$:

$$\frac{\partial V^v(j,\mathbf{Z})}{\partial \mathbf{z}_j} = \boldsymbol{\zeta} + \beta \frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^v(j,\mathbf{Z}')|\mathbf{Z}\right] + \underbrace{\beta\delta^v\gamma\sum_{j'=1}^{J}\mu_{j'}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))\frac{\partial\nabla(j,\mathbf{Z})}{\partial \mathbf{z}_j}}_{\text{Opportunities to sell the vacant unit}} \tag{16}$$

where $\boldsymbol{\zeta}$ is renters' preference for amenities. The owner of a vacant house in a neighborhood with a more favorable index of trading opportunities $\nabla(j, \mathbf{Z})$ will turn a greater transaction surplus when selling the vacant unit.

Combining the gradient of the value of living-in (15) and that of owning a vacant unit (15) into (14) yields the price amenity-gradient as the sum of the current utility flows of owning a house, the impact of current amenities of the future values of owning a house, and the capitalization in the price of the change in the trading opportunities with respect to amenities.

Proposition 3. (Impact of trading opportunities on price-amenity gradient) *The relationship between log prices and amenities is driven by the neighborhood's index of trading opportunities ∇. The price-amenity gradient is the sum of three terms:*

$$E\left[\frac{\partial \log \mathbf{p}_{\cdot j}}{\partial \mathbf{z}_j}\right] = \boldsymbol{\chi} + \overbrace{\gamma\beta\frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^\ell|\mathbf{Z}\right] + (1-\gamma)\beta\frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^v|\mathbf{Z}\right]}^{\textit{Impact of current amenities on future values}} + \underbrace{Capitalization_j \cdot \frac{\partial \nabla}{\partial \mathbf{z}_j}(j,\mathbf{Z})}_{\textit{Impact on trading opportunities}} \tag{17}$$

where $\boldsymbol{\chi}$ is the weighted average of owners' and renters' preferences $\boldsymbol{\chi} = \gamma\boldsymbol{\xi} + (1-\gamma)\boldsymbol{\zeta}$. The average of the capitalization coefficients across neighborhoods is equal to $E(Capitalization_j) = \frac{H}{L}(1 - \frac{1}{2}\sum_{j=1}^{J}\left(\frac{h_j}{H}\right)^2) - 1$. This is positive. This coefficient declines when the number of listings increases.

Proof. See Appendix Section D.3. □

The proposition implies that when neighborhoods with higher amenities have worse trading-up opportunities ($\partial \nabla / \partial \mathbf{z}_j < 0$), the price-amenity gradient $\partial \log \mathbf{p}_{kj} / \partial \mathbf{z}_j$ is flatter than what the vector of preferences for amenities $\boldsymbol{\chi}$ suggest for given expectations of the impact of current amenities on the future value of owning a house. In this case, the household faces a trade-off between high current value of owning a house with low trading opportunities, and low current value of amenities with high trading opportunities. In the opposite case, trading opportunities are aligned with the current value of amenities and the price-amenity gradient is steeper than what the vector of preferences for amenities $\boldsymbol{\chi}$ suggests. In absence of credit frictions, the capitalization coefficient is constant and only depends on aggregate ratio of housing units to listings and on the distribution of housing units across the city. As we shall see in Section 3.3 this will no longer be the case when time-varying credit frictions are introduced. In this case, changes in credit frictions will affect both the trading opportunities-amenities gradient and the capitalization of trading opportunities.

3.2 Arbitrage Opportunities

The city-wide heterogeneity in trading opportunities $\nabla(j, \mathbf{Z})$ is a reflection of the heterogeneity in trading-up opportunities across locations that would not occur in a frictionless Alonso et al. (1964) city. In such a frictionless world, described in Appendix Section (D.4), log prices would indeed reflect (i) the value of amenities and (ii) housing supply. The Alonso et al. (1964) case obtains with an infinitely elastic housing supply.

We thus consider briefly the introduction of a real estate investor with access to a perfect matching technology, i.e. which can choose freely which sellers and buyers he can be matched to. Such a real estate investor can buy a vacant home in neighborhood k at any price exceeding $V^v(k, \mathbf{Z})$. He can sell such vacant home to a buyer from neighborhood j at a maximum price of $V^\ell(k, \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^\ell(j, \mathbf{Z})$. Thus the maximum profit that an arbitrageur can earn is the maximum over $(k, j) \in \{1, 2, \ldots, J\}^2$ of the difference $V^\ell(k, \mathbf{Z}) + V^v(j, \mathbf{Z}) - V^\ell(j, \mathbf{Z}) - V^v(k, \mathbf{Z})$, which is simply the difference in trading opportunities $\nabla(k, \mathbf{Z}) - \nabla(j, \mathbf{Z})$,

$$\Pi = \max_{k,j} \{\nabla(k, \mathbf{Z}) - \nabla(j, \mathbf{Z})\} \tag{18}$$

In other words, the arbitrageur will buy a vacant home with the lowest amenity-price differential and sell it to a buyer from the neighborhood with the highest amenity-price differential.

With a single arbitrageur, this will not affect other households' outside option. Nevertheless, with a large number of arbitrageurs, arbitrageurs will compete for vacant houses, and buyers will compete among multiple arbitrageurs, and the free-entry, zero profit condition for arbitrageurs will lead to a homogenous trading opportunities index, i.e. $\forall j\, \nabla(j, \mathbf{Z}) = \nabla(\mathbf{Z})$ across neighborhoods. The following proposition formalizes this result:

Proposition 4. (Impact of arbitrageurs on prices) *In a city with free entry of arbitrageurs with no matching frictions, the index of trading opportunities is constant across locations. The zero-profit condition implies that the index of trading opportunities does not vary across neighborhoods,* $\nabla(j, \mathbf{Z}) = \nabla(\mathbf{Z})$. *Then the log price exhibits a linear relationship with the preference coefficient* $\boldsymbol{\xi}$, *and trading-up opportunities* ∇ *do not affect price-amenity gradients differently across locations.*

The last statement is a simple consequence of expressions (15) and (16).

3.3 Price-Amenity Gradient with Credit Frictions

The city with matching frictions predicts a constant relationship between prices, amenities, and the index of trading opportunities ∇. In contrast, as lending standards fluctuate over time, shifts in credit frictions alter the relationship between prices and the trading opportunities index. When lending standards are tight, trading-up opportunities are limited and the index of trading opportunities ∇ has little impact on the price-amenity gradient. When lending standards are more relaxed, trading-up opportunities are more readily available and the index of trading opportunities ∇ has a larger impact on the price-amenity gradient.

The case of a uniform approval rate for mortgage applications can help clarify this mechanism. The average amenity-price gradient across neighborhoods becomes:

$$E_j\left[\frac{\partial \log \mathbf{p}_{kj}}{\partial \mathbf{z}_j}\right] = \boldsymbol{\chi} + \overbrace{\gamma\beta\frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^{\ell}(j, \mathbf{Z}')|\mathbf{Z}\right] + (1-\gamma)\beta\frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^{v}(l, \mathbf{Z}')|\mathbf{Z}\right]}^{\text{Impact of current amenities on expectations of future amenities}}$$
$$\underbrace{+\, Capitalization(\varphi_j, B_j, \kappa_j)\cdot\frac{\partial \nabla}{\partial \mathbf{z}_j}(j, \mathbf{Z})}_{\text{Impact on trading-up opportunities}}. \tag{19}$$

Hence the capitalization coefficient depends on the mortgage approval probability φ_j, the outstanding mortgage balance B_j, and the interest cost κ.

The following proposition suggests that the capitalization coefficient declines when the approval rate φ declines, when the outstanding mortgage balance B increases, and when the interest cost increases κ.

Proposition 5. (Partial Equilibrium Impact of Credit Frictions) *Keeping the trading opportunities, credit conditions, and listings constant in all other neighborhoods, a tightening in credit conditions in one neighborhood (i.e. a reduction in the approval rate, an increase in the mortgage balance, or an increase in the interest cost) reduces trading opportunities, the capitalization coefficient, and the number of listings in that neighborhood.*

The logic of the proof is described in Figure 4 for an increase in mortgage balance in one neighborhood. Figure (a) describes the determination of the equilibrium local trading opportunity index as the fixed point of the Bellman equation (29). An increase in the mortgage balance implies that more heavily indebted households will be less able to seize future trading-up opportunities, which, in turn, reduces the number of listings in this specific neighborhood and the opportunity to sell vacant units to households from other neighborhoods. This causes a decline in the neighborhood's index of trading opportunities.

Figure (b) graphs the relationship between the neighborhood's index of trading opportunities and the capitalization coefficient. A higher balance leads to a lower capitalization at every given level of the index. For a giving set of trading opportunities, the tightening of credit constraints reduces the set of potential trades that generates a surplus for buyers and sellers. This reduces the probability to sell and therefore the capitalization of trading opportunities into prices.

4 Identification

4.1 The Model without Credit Frictions

We present here first the estimation of the index of trading opportunities ∇_{jt} for each neighborhood and each year and the values V_{jt}^{ℓ} and V_{jt}^{v}. Then using such estimate, we estimate preferences for amenities $\boldsymbol{\xi}$, the intertemporal discount factor β, and the variance of unobservables σ_{ε}^{2} .

Step #1: Identification of Trading Opportunities

The data set has $t = 1, 2, \ldots, T$ years. The indexes of trading opportunities ∇_{jt} are identified by observing gross flows *into* each neighborhood j in each period t. For the neighborhoods j for which the number of movers is positive, $n^+_{jt} > 0$, the vector $\boldsymbol{\nabla}_t$ of trading opportunities is such that the predicted probability of moving to j' is equal to the *observed* probability of moving to j':

$$\log\left[\sum_{j=1}^{J} n_{jt}\delta\lambda_{j'}F\left(\nabla_{j't}-\nabla_{jt}\right)\right] = \log n^+_{j't}, \tag{20}$$

The vector $\boldsymbol{\nabla}_t$ is obtained by minimizing the distance between the prediction and the realization of mobility. If the probability of moving in to a neighborhood is $n^+_{jt} = 0$, the index of trading opportunities of this neighborhood is conventionally set to $\nabla_{j't} = -\infty$.

Step #2: Identification of Values

Observed prices are then separately identifying the utilities V^ℓ of living in a neighborhood and V^v of owning a vacant unit. Indeed, prices depend on the sum $\gamma V^\ell_{j't} + (1-\gamma)V^v$,

$$\log p_{jj't} = \gamma \cdot [V^\ell_{j't} - \nabla_{jt}] + (1-\gamma)\cdot V^v_{j't}, \tag{21}$$

then observation of prices $\log p_{jj't}$ and identification of ∇s using flows provides two relationships to identify $V^\ell_{j't}$ and $V^v_{j't}$. However, the econometrician does not typically observe $\log p_{jj't}$. Transaction data features the destination of sales but not the origin of sales. To solve this we estimate the average index of trading opportunities $E(\nabla_{jt}|j')$ of buyers moving in to each neighborhood j', from any origin j. The probability that a sale in neighborhood j' is with a buyer living in j is simply, by Bayes' law:

$$P(j|j') = \frac{P(j'|j)P(j)}{\sum_{k=1}^{J} P(j'|k)P(k)} = \frac{\lambda_{j'}F_{jj't}n_{jt}}{\sum_k \lambda_{j'}F_{kj't}n_{kt}}, \tag{22}$$

where $F_{jj't} = F(\nabla_{j't} - \nabla_{jt})$ is the probability of a transaction conditional on a match between a buyer in j and a seller in j'. Thus the average index $\nabla_{\cdot t}$ of buyers in j' is $E(\nabla_{\cdot t}|j') = \sum_{j=1}^{J} P(j|j')\cdot \nabla_{jt}$. Therefore:

$$\gamma \cdot V^\ell_{j't} + (1-\gamma)\cdot V^v_{j't} = \underbrace{E_j \log p_{jj't}}_{observed\,price} + \gamma \cdot E(\nabla_{\cdot t}|j') \tag{23}$$

Together, the J moment conditions of flows (20) and of prices (23) identify the $2J$ values V^{ℓ}_{jt} and V^{v}_{jt}. Sellers' bargaining power is set to $\gamma = 1/2$, as is conventional in part of the search and matching literature (Den Haan, Ramey & Watson 2000).[4]

Step #3: Identification of the Arrival Rate of Offers

The arrival rate of offers is estimated by matching the predicted population change with the observed population change Δn_{jt}.

$$\hat{\delta} = \text{argmin} \sum_{j,t} \Bigg[\Delta n_{jt} - \hat{\delta} \Bigg\{ \ell_{jt} \sum_{j'=1}^{J} \mu_{j't} F \left[\nabla_{jt} - \nabla_{j't} \right] . \\ - n(j, \mathbf{Z}) \sum_{j'=1}^{J} \lambda_{j't} F \left[\nabla_{j't} - \nabla_{jt} \right] \Bigg\} \Bigg]^2 \tag{24}$$

Step #4: Identification of Preferences

The final step identifies the vector of preferences for amenities $\boldsymbol{\xi}$ and the intertemporal discount factor β. Noting $\mathbb{E}_{jt+1}$ the expectation of the value of living, from neighborhood j in period $t+1$, the following regression identifies preference coefficients $\boldsymbol{\xi}$ and the time discount factor β:

$$V^{\ell}_{jt} = \mathbf{z}_{jt} \boldsymbol{\xi} + \beta \mathbb{E} \left[V^{\ell}_{j'} \right] + \varepsilon_{jt}, \tag{25}$$

which is identified whenever unobservables are orthogonal to observable amenities and to the expectations. $\mathbf{z}_{jt}$ includes a neighborhood fixed effect.

This regression requires an estimate of the expectation term, in a way akin to a control function approach. Expectations depend on (i) the probability of being matched to a vacant house δ, (ii) conditional on a match, the probability $\lambda_{j't}$ that this match is in neighborhood j'; the listings are observed and thus $\lambda_{j't}$ too; (iii) the expected value V^{ℓ} of living in the neighborhood in subsequent periods.

As the future evolution of the neighborhood may depend on values of amenities that are never observed, we estimate a flexible relationship between values and amenities noted $V^{\ell}(\mathbf{z}, \varepsilon, \mathbf{Z})$, i.e. it depends on the observable amenities of the neighborhood, the unobservable amenities ε_{jt}, and the

[4] There is spatial and time variation in sellers' share of the transaction surplus as sellers' and buyers' outside options are heterogeneous.

distribution of amenities $\mathbf{Z}$ in the city. We estimate such relationship using a flexible Generalized Additive Model (Hastie & Tibshirani 1990). This enables the estimation of values of indirect utilities outside of the empirical support of $\mathbf{z}_{jt}$ and the estimation of the expectation:

$$\mathbb{E}(V_{j'}^{\ell}) = \int_{\mathcal{E}} \int_{\mathcal{Z}} V^{\ell}(\mathbf{z}', \varepsilon', \mathbf{Z}') f(\mathbf{z}', \varepsilon', \mathbf{Z}' | \mathbf{z}, \varepsilon, \mathbf{Z}) d\mathbf{z} d\varepsilon \tag{26}$$

where the dynamic of neighborhood amenities $\mathbf{z}'|\mathbf{z}$ is estimated using the neighborhood panel, and ε_{ijt+1} is an iid shock with normal distribution.

An autoregressive process is estimated for each amenity $\mathbf{z}$. This expectation 26 is estimated by Monte Carlo integration by drawing values $(\mathbf{z}, \varepsilon, \mathbf{Z})$ from this AR process conditional on the previous values of the amenities. The unobservables ε' in the next year are assumed independent of the unobservables in the previous year.

This enables the estimation of preferences $\boldsymbol{\xi}$ and the time discount factor β by estimating specification (25).

Individual Heterogeneity

For the sake of clarity, identification was presented in this section without indexing utilities by household i. Yet, a similar approach estimates values by demographic subgroup. Each individual i is characterized by a set of K demographic characteristics $\mathbf{x} \in \mathbb{R}^K$, such as income, race, and ethnicity. The identification uses the share of movers of each characteristic $\mathbf{x}$ in each neighborhood j. This provides the trading opportunities index $\nabla_{jt}(\mathbf{x})$, and the values $V_{jt}^{\ell}(\mathbf{x})$, $V_{jt}^{v}(\mathbf{x})$. This enables an estimation of heterogeneous preferences $\boldsymbol{\xi_i} = \boldsymbol{\xi}(\mathbf{x}_i)$ and heterogeneous time discount factors $\beta_i = \beta(\mathbf{x}_i)$.

4.2 Identification with Credit Frictions

Credit frictions affect each of the four steps of the estimation.

In the first step, the predicted gross flow of movers is affected by the mortgage balance, the interest cost, and the approval rate:

$$\log\left[\lambda_{j'}\sum_{j=1}^{J} n_{jt}\cdot\varphi_{j't}\cdot F\left(\nabla_{j't}-\kappa_{j't}-B_{jt}-\nabla_{jt}\right)\right]=\log n^{+}_{j't} \tag{20'}$$

where (i) $\varphi_{j't}$ is the mortgage approval probability in j'; a low mortgage approval probability leads to low inflows; (ii) B_{ijt} is the mortgage balance at the origin; high indebtedness lead to low outflows from j; (iii) $\kappa_{ik't}$ is the interest cost; high interest costs lead to low inflows into j'. The approval probabilities, mortgage balances, and interest costs are observed and presented in Section (5) below.

In the second step, values V^{ℓ} and V^{v} are also affected by the mortgage balance and the interest cost. Indeed, the mortgage balance and the interest cost affect the value of moving for the buyer, and thus the negotiated price (equation). Thus the sum $\gamma\cdot V^{\ell}_{ij't}+(1-\gamma)\cdot V^{v}_{ij't}$ of the values becomes:

$$\gamma\cdot V^{\ell}_{j't}+(1-\gamma)\cdot V^{v}_{j't}=\underbrace{E_j\log p_{ijj't}}_{observed\,price}+\underbrace{\kappa_{ij't}}_{interest\,cost}+\gamma\cdot E_j\left(\nabla_{ijt}+\underbrace{B_{ijt}}_{balance}\right), \tag{21'}$$

which, together with the index of trading opportunities, yields values V^{l}_{ijt} and V^{v}_{ijt} that account for borrowing constraints. Adjustments to the third and fourth estimation steps follow from these modifications.

5 Data Set

Five data sets are combined to estimate the model. The data construction steps are detailed in Appendix Section B.

First, flows into each neighborhood (identification step #1) are obtained using a comprehensive data set of all deeds transactions from the 14 county recording offices of the Chicago metropolitan statistical area between 2005 and 2014 inclusive. The data includes the street address of the transacted property, the unique assessment parcel number (APN), the nature of the transaction (grant deed or deeds of trust for typical transactions), the transaction amount, the name of the buyer, and the name of the seller. The transaction amount is used in identification step #2. The race, ethnicity, and income of the buyer is obtained by matching deeds to data from the Home Mortgage Disclosure Act. The HMDA is matched to deeds using the location of the property and the loan amount.

Second, the deeds data include the loan amount, the structure of the mortgage (ARM or FRM, negative amortization or interest-only loan, loan maturity), the name and address of the lender. We also observe mortgage refinancings and second mortgages. This provides an estimate of mortgage loan balances, interest cost, and mortgage payments using mortgage amortization formulas (Brueggeman & Fisher 2011), for each Assessment Parcel Number for which there is at least one transaction in the files. Approval rates are obtained using the universe of mortgage applications of the Home Mortgage Disclosure Act.

The third set of data pertains to neighborhood amenities: schools, crime, and neighborhood demographics. Such observations enable the identification of households' preferences for amenities (identification step #4).

The median and average school test score measures are built as follows. The 3,057 schools are matched to their corresponding school-attendance areas, within the 368 school districts. Each blockgroup is matched to its school-attendance area and linked to the average test score of the attendance area. School test scores are provided by each of the three states: in Illinois, the *State Board of Education*'s *Report Card with Assessment Data;* in Wisconsin, the Department of Public Instruction provides data from the *Wisconsin Student Assessment System*; in Indiana, the Department of Education provides *Annual School Performance Reports* ranking schools with letters from A to F.

Crime data at the neighborhood level is based on the Federal Bureau of Investigation's *Uniform Crime Reports* and from the City of Chicago's Crime Data Portal. Each blockgroup is matched to either a corresponding incorporated municipality or a county, when outside an incorporated municipality. Crime counts in blockgroups within incorporated municipalities are the crime counts of the municipal police. Crime counts in neighborhoods within a county outside the boundaries of an incorporated municipality are those of the County Sheriff.

Neighborhood data comes from the American Community Survey (ACS). At the blockgroup level, and between 2005 and 2016, the ACS provides 5-year averages for key household demographics and for housing units: median household income, median building age, median number of rooms, racial and ethnic make-up. Such 5-year averages overlap as the 5-year window shifts by one year every year. In a similar fashion as in Autor, Dorn & Hanson (2013), there are thus a set of accounting relationships that provide a set of constraints on each neighborhood-level annual measure. Using a flexible location-specific polynomial it is then possible to recover estimates of the annual, blockgroup-level data. Such data includes the age of buildings, the median number of rooms and the demographic make-up of the neighborhood (race, ethnicity, income, and education). Neighborhood demographics are used in identification step #3 (the arrival rate of offers) and step #4 (identification of the preference for neighbors' demographics).

6 Empirical Analysis

The goal of this section is to present a series of key empirical and model-derived facts on the evolution of the Chicago MSA between 2007 (the final year of the boom) and 2010 (the trough of the mortgage crisis). Such facts highlight the role of time-varying credit frictions in the joint dynamics of prices and trading opportunities.

Section 6.1 below presents the outcome of the estimation of the dynamic model. In particular it regresses values and trading opportunities on local amenities. Sections 6.2 and 6.3 show how, at constant preferences, shifts in credit conditions can lead to large changes in the relation between prices and amenities. Section 6.4 presents a test of this paper's theory-driven decomposition of the price-amenity gradient into preferences and trading opportunities (Equation 3.3).

The correlation between credit conditions and the stark inequalities in crime rates and school quality across the Chicago metro area imply a time-varying pricing of amenities.

6.1 Estimated Preferences and Trading Opportunities

Table 1 presents the estimation of preferences and the time discount factor for the model with credit (col. 1-2), and for the model without credit (col. 3-4).

The first result is that for both models the expectation $\mathbb{E}\left[V^{\ell}(j', \mathbf{Z}')|j, \mathbf{Z}\right]$ is highly significant, confirming the role of dynamic considerations in housing preferences. The model with credit yields a higher estimate discount factor than the model without credit (0.58 vs. 0.52) which is consistent with levered borrowers being more concerned with future housing valuations. The second result is that accounting for credit has a significant impact on the valuation of several amenities. In particular, taking into account credit conditions reveals a much stronger preference by ethnic groups for living in neighborhoods with a large fraction of neighbors of the same race. This is true for Blacks, Hispanics, and Asians. The model with credit also predicts stronger preferences for larger and more recent houses.

Underlying the differences between the estimates of the two models is the idea that accounting for credit enables to disentangle the taste for neighborhoods based on preferences from considerations related to access to credit or to the cost of borrowing. For example, Blacks might have a strong preference for mostly blacks neighborhoods but the model without credit yields a weaker preference because of the difficulties of Blacks to obtain credit to live in such neighborhoods. By contrast, the model with credit produces households preferences that are purified of credit considerations.

The distribution of trading opportunities across neighborhoods with different amenities has implications for the relationship between prices and amenities (see Proposition 3). Here again, there are substantial differences between the estimates of the two models. The model with credit predicts a distribution of trading opportunities who mostly mirror the preference for amenities. This is the less the case for the model without credit for which trading opportunities decline in the fraction of Blacks both for Blacks and for other ethnic groups.

6.2 Price Dynamics, Credit Frictions, and Trading Opportunities

Figure 1, panel (a) and panel (b), looks at the price-amenity gradient for two key amenities: (i) school quality, measured by the test-score (z-score), and (ii) crime measured by the number of offenses per capita. Neighborhoods are sorted by bins of increasing z-score with equal numbers of neighborhoods in each bin. For both amenities, the price-amenity gradient is much *steeper* in 2010 than in 2007.

Figure 1, panel (c) looks at the spatial distribution of price changes in City of Chicago. It shows an important contrast between the strong price declines between 2007 and 2010 in the south and west sides and the more moderate price changes in the rest of the city. Figure 1, panel (d) plots the 2005 school test-scores, ranked by quartiles of the neighborhood distribution of z-score on the same map. The map reveals that those neighborhoods experiencing large price drop are also neighborhoods with below average school quality. The steepening of price-school test score gradient (panel (a)) thus results, for a large part, from larger house price declines in neighborhoods of the south and west side of the City of Chicago, which are at the bottom end of the distribution of school test scores.

The next step is to assess which of the dimensions of credit conditions are driving such decline in house prices in the West and South Side: (i) the mortgage approval rate, (ii) households' mortgage balance, (iii) the interest cost on new mortgages. Figure 2 looks at the time variation in the correlation between credit conditions and neighborhood amenities between 2007 and 2010. Figure 2 panel (a), (b), and (c) display the average mortgage balance over current price (in logs), the average present value of mortgage interest cost over current prices (in logs), and the average approval rate of mortgage applications, respectively. A striking fact is that neighborhoods with the lowest test scores experienced the largest increase in average mortgage balance between 2007 and 2010. In 2010, the average ratio mortgage balances is above the current house price (share above one) for the neighborhoods that are at the bottom third of the school quality distribution. This fact implies that, on average, households in those neighborhoods have "underwater" mortages in 2010. By contrast the interest cost remains very similar in both periods, with higher risk premia being compensated by lower base interest rate. The approval rate for new mortgages increased in most neighborhoods, with fall in house prices more than compensating more stringent lending standard in the context of a collapse in the number of applications (this is addressed in Appendix Section C whose results are displayed in Appendix Figure C). Figure 2, panel (b) looks at the change in household income across the distribution of neighborhoods, suggesting little change. In sum, the sharp deterioration in mortgage balances in the neighborhoods with low school quality appears as the most salient sign of the uneven deterioration in credit conditions.

Figure 3 displays a map of the distribution of mortgage balances across the neighborhoods of Cook County, Illinois. Average mortgage balances over current house prices are ranked by quartile computed over the 2005-2013 period. The fourth quartile includes neighborhood whose mortgage balances are over 95 percent of the house price (the median balance in that quartile is 143 percent) and thus characterizes areas in which a vast fraction of mortgages are "underwater". Comparing panel (a) for 2007 and panel (b) for 2010 shows a large increase in mortgage balances throughout the County but strong spatial contrasts. The neighborhoods that are affected by the most dramatic increase in mortgages balances are in the South Side and the West Side of Chicago – the same neighborhoods in which school test scores are lower – as well as cities adjacent to the South Side (e.g. Harvey) which are also typically characterized by low levels of school test scores.

Figure 5 maps the spatial distribution of trading opportunities in Cook County in 2007 and

2010. Against the backdrop of a general reduction in trading opportunities between 2007 and 2010, the most dramatic fall in trading opportunities appears in the same areas of Cook County where school test scores are low and mortgage balances were high during the crisis.

Our theoretical framework (Figure 4) predicts that tighter credit frictions lead to both a fall in trading opportunities and in their capitalization into prices. When this fall happens in neighborhoods with a low quality of amenities, which is shown in Figures 2 and 3, the theory predicts a steepening of the price-amenity gradient, which is what is observed in the data for Chicago (Figure 1).

6.3 The Price-Amenity Gradients: Preferences vs. Trading Opportunities

A key result of our model regards the impact of trading opportunities on the price-amenity gradient. The theory Section predicts that the price-amenity gradient captures both preferences, expectations, and trading opportunities 3. In our model, with stable preferences, changes in price-amenity gradients $\frac{\partial \log \mathbf{p}_{kj}}{\partial \mathbf{z}_j}$ are correlated with changes in the gradient $\frac{\partial \nabla(j,\mathbf{Z})}{\partial \mathbf{z}_j}$ of trading opportunities with respect to amenities. As a way to to assess this relationship, we first run the following regression of log prices on amenities interacted with each year indicator, controlling for a neighborhood (blockgroup) fixed effect and year fixed effect:

$$\log p_{jt} = \sum_{t=2005}^{2013} \alpha_t \cdot (\text{Year}_t \times \mathbf{z}_{jt}) + \text{Neighborhood}_j + \text{Year}_t + \varepsilon_{jt}. \tag{27}$$

We then re-run the same regression controlling for the trading opportunities. The regression is done for four different neighborhood characteristics: fraction of African Americans, fraction of Hispanics, crime (log of the number of offenses per capita), and school quality (z-Score).

Figure 7 reports the coefficients of the regression of the log transaction price in each neighborhood on each of its four characteristics, interacted with an indicator variable for each year. These coefficients, linked on Figure 7 by a solid line, measures the price-amenity relationship each year between 2005 and 2013. A second set of coefficents, linked with a dotted line, measures the price-amenity relationship each year between 2005 and 2013 *controlling for trading opportunities.* There is a striking contrast between the two set of estimated coefficients. Without controlling for trading opportunities, the price-amenity gradient displays significant time-variations. For the fraction of

African Americans (upper-left panel), the coefficient is *positive* in 2005-2009, but then becomes negative in 2010-2013. For the fraction of Hispanics, the coefficient similarly turns from *positive* in 2005-2009, to negative in 2010-2012. The lower-right panel shows that for the standard school test score in Illinois, the coefficient goes from *negative* in 2005-2008 to positive in 2009-2013. While the lower-left panel for log(offenses per capita) is less clear, it nevertheless suggests substantial variation in the capitalization coefficient over the cycle. By contrast, when trading opportunities are controlled for, the price-amenity relationship becomes almost stable over time, suggesting that variations in trading opportunities can indeed account for most of the variation in prices-amenity gradients.

Figure 8 plots the coefficients of the regression of the trading opportunities index ∇ on the same set of neighborhood characteristics, interacted with years, and controlling for block-group fixed effects. The changes in trading opportunities are following a similar pattern than those in price-amenity gradients when the trading opportunities are not controlled for (Figure 7). Figure 8 shows a shift in the location of trading opportunities between the boom and the bust away from neigborhoods with high fraction of minorities and below average school quality and towards neighborhood to neighborhoods with lower shares of minorities and higher school quality. This shift, whose spatial dimension is illustrated on Figure 5, explains while preferences only become stables when trading opportunities are controlled for.

Figure 6 complements Figure 8 by plotting the estimates of the impact of changes in trading opportunities over changes in mortage balances, showing that, as predicted by the theory's Proposition 5, trading opportunities sharply decrease when credit conditionss tighten.

Table 2 decomposes the reversal of the price-school quality gradient by looking at the role of time-variations in transaction prices, trading opportunities, and future expected value of living in the neighborhood. Each regression controls not only for blockgroup fixed effects but also for the entire arrays of neighborhood characteristics. Table 2 focuses on school quality but similar results are obtained for other neigborhood characterisitics. The table documents the reversal in the capitalization of school quality between the boom and the bust and shows that it closely matche the reversal in trading opportunities. By contrast the future expected value of living in the neighborhood appears rather stable over time.

In short, this subsection illustrates the importance of trading opportunities in the dynamics of the price-amenity gradient. Trading opportunities appear to be the key ingredient necessary to reconcile time-variations in the price-amenity gradient with stable current and future preferences over amenities.

6.4 Credit Frictions and the Capitalization of Trading Opportunities

Our theoretical results predict that a tightening of credit friction not only reduce trading opportunities but also reduce the capitalization of trading opportunities into prices by reducing the potential surplus from house transactions. We evaluate here the impact of credit frictions on the capitalization coefficient within a regression framework that is exactly derived from the the theoretical price-amenity gradient in the presence of credit frictions (Equation 19). We run the following regression:

$$\begin{aligned}\log p_{jt} = a \cdot \mathbf{z}_{jt} + b \cdot \nabla_{jt} + c \cdot \nabla_{jt} \cdot Credit_{jt} + d \cdot Credit_{jt} \\ + e \cdot \mathbb{E}\left(V^{\ell}_{jt+1}\right) + Neighborhood_{jt} + Year_t + \varepsilon_{jt},\end{aligned} \tag{28}$$

where $\mathbf{z}_{jt}$ is the vector of neigborhood characteristics, ∇_{jt} the trading opportunities, $Credit_{jt}$ the three measures of credit frictions (approval rate, interest rate cost, mortgage balances), and $\mathbb{E}\left(V^{\ell}_{jt+1}\right)$ the future expected utility of living-in the neighborhood. The trading opportunities index is obtained in Identification Step #1 (4). Expectations are obtained by building transaction probabilities and integrating values over the distribution of future amenities (Step #4, equation 26 with the adjustment for credit frictions described in Section 4.2).

Table 3 present regression results where the different components are graduallly introduced. Table 3, col 1., presents, as a baseline, the results of standard hedonic regression without trading opportunities and expected utility of living-in which are then introduced in Table 3, col 2. Table 3, col 3. adds the interaction of trading opportunities with credit frictions. Trading opportunites and the expected value of owner-occupying are strongly significant. Introducing those terms affect some of the amenity coefficients with the mean test score (IL) displaying the expected positive sign and the negative impact of crime being reduced. The index of trading opporunities is standardized with a unit variance. A one standard deviation increase in trading opportunities is associated with an increase of 16.9 percent in house prices. Table 3, col 3. shows that the coefficient capitalization significantly varies with approval rates and mortgage balances. The effect is economically large. A increase in mortgage balances from 80 to 100% of the current price lowers the capitalization coefficient by 0.06 which is more than a third of the baseline capitalization coefficient of 0.169 (Table 3, col. 2.)

The results of Table 3 complete the analysis of the price-amenity gradient relationship by showing that, as the theory predicts, tighter credit constraints not only reduce the emergence of trading opportunities (Figure 6) but also their capitalization into house prices.

7 Conclusion

This paper shows that variations in trading opportunities combined with variations in the capitalization of trading opportunities are able is to explain the large observed variations in the pricing of amenities in absence of variations of preferences. Time variation in housing search and credit market frictions are able to explain, in turn, why trading opportunities and their capitalization into prices change over time. During the mortgage boom, easy access to mortgage credit facilitated the exploitation of trading opportunities in high amenity neighborhoods, shifting trading opportunities to neighborhoods experiencing gentrification. Furthermore when credit conditions are relaxed or when the supply of listing is large, trading opportunities are easier to grasp and therefore neighborhoods with a high level of trading opportunities are traded at a lower premium than when market frictions are more severe.

The paper's results have substantial implications for both policymakers and analysts of housing markets. Measuring the dollar willingness-to-pay for public goods is a task that relies on house prices to identify households' preferences for amenities. Understanding the elasticity of housing demand with respect to amenities relies on an understanding of the structural drivers of the house price-amenity relationship. This paper proposes a framework combining the estimation of preferences, the dynamics of amenities and that of trading opportunities which match key empirical facts on the time variation of the house price-amenity relationship. In estimating the relationship between prices and observable characteristics, machine learning approaches recently pioneered in economics (Mullainathan & Spiess 2017) are susceptible to a form of Lucas critique (Lucas 1976), as the relationship between preferences and house prices depends on the organization of the market, e.g. the nature of its frictions. Hence even econometricians interested in the task of *predicting* rather than explaining house prices would benefit from using a structural framework such as ours, in which forward-looking households make housing decisions by taking into account the dynamics of both amenities and market frictions.

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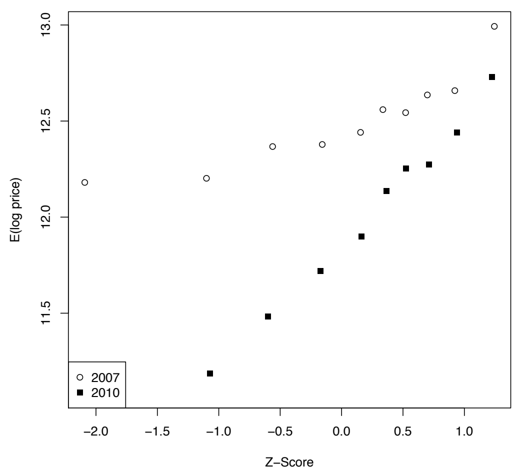
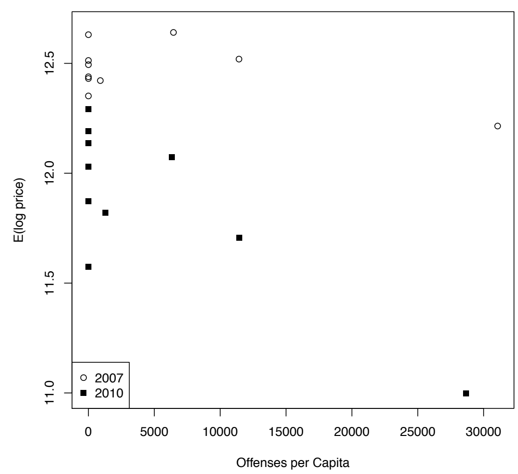
Figure 1: Descriptive Facts – Price Dynamics in the Chicago Metro Area

This set of figures presents the evolution of the correlation between prices and amenities in the Chicago Metro Area between 2007 and 2010. Figures (a) and (b) suggest that the correlation between school test scores (resp., offenses per capita) and house prices became substantially steeper during the bust, in 2010. Figure (c) shows that the largest price declines were concentrated in the southern part of the City of Chicago. Figure (d) suggests that the distribution of house prices expanded, with larger declines at the bottom of the price distribution.

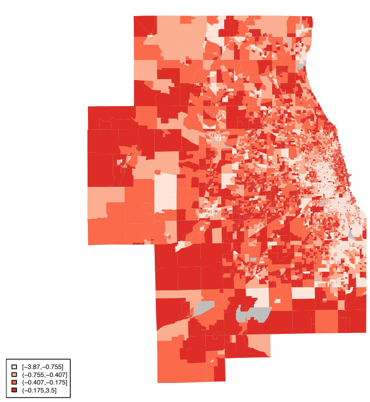
(a) Average Price by Std. Test Score *(Illinois)*



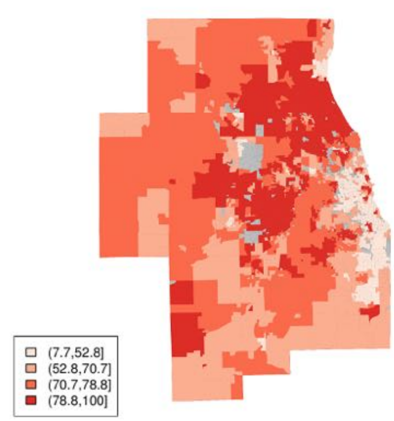
(b) Price and Crime Rate (*Chicago-Naperville-Elgin MSA)*



(c) Price Declines 2007-2010



(d) Average Test Scores in 2005

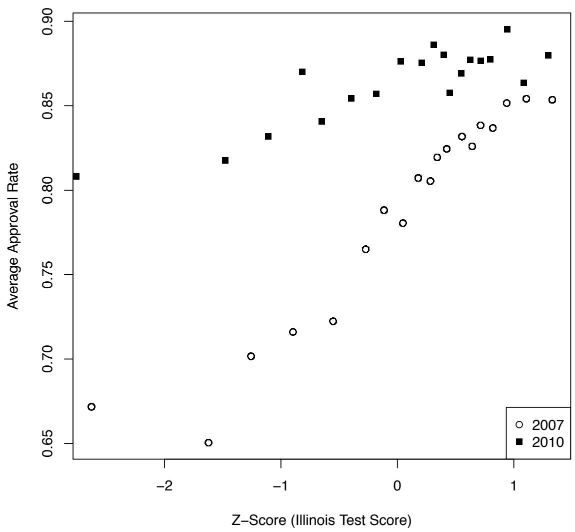


Sources: City of Chicago geocoded criminal incidents. FBI's Uniform Crime Reports. Illinois State Board of Education. House prices from Corelogic's deeds records. School attendance areas hand coded.

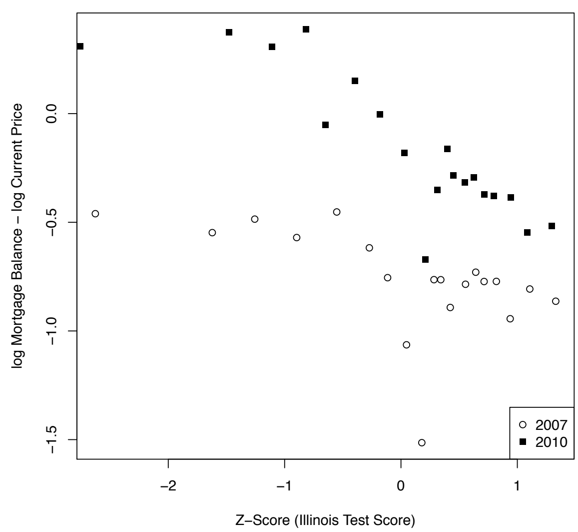
Figure 2: The Correlation of Amenities with Credit Frictions and Household Income

This figure presents the correlation between three dimensions of credit frictions and the school attendance areas' z-score, for 2007 and 2010. Neighborhoods are sorted by bins of increasing z-score with equal numbers of neighborhoods. Each point presents the average approval rate, log mortgage balance as a share of the current price, log interest cost as a share of the current price. Figure (d) presents household income by z-score bin in 2007 and 2010.

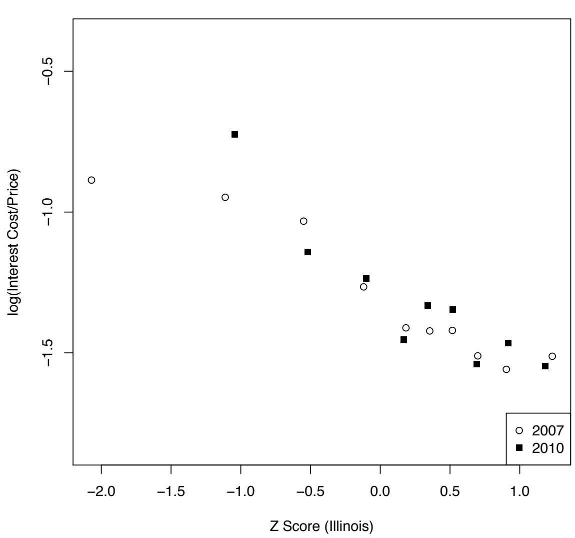
(a) Approval Rate and Z-Score



(b) Mortgage Balance and Z-Score



(c) Interest Cost and Z-Score



(d) Household Income and Z-Score

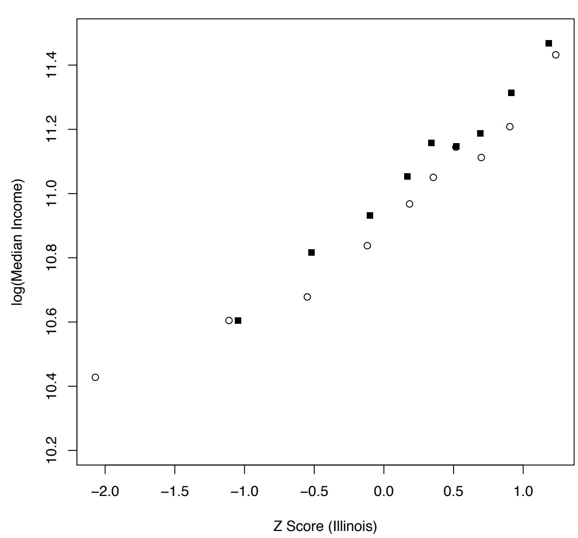


Figure 3: The Spatial Distribution of Mortgage Indebtedness – Cook County

This map focuses on the neighborhoods of Cook County, Illinois. It presents the evolution of the spatial distribution of mortgage indebtedness. Mortgage indebtedness is the log of the remaining mortgage balance over the current transaction price of housing units. The map illustrates the substantial increase in indebtedness in the south relative to the northern parts of the county.

(a) 2007 (b) 2010

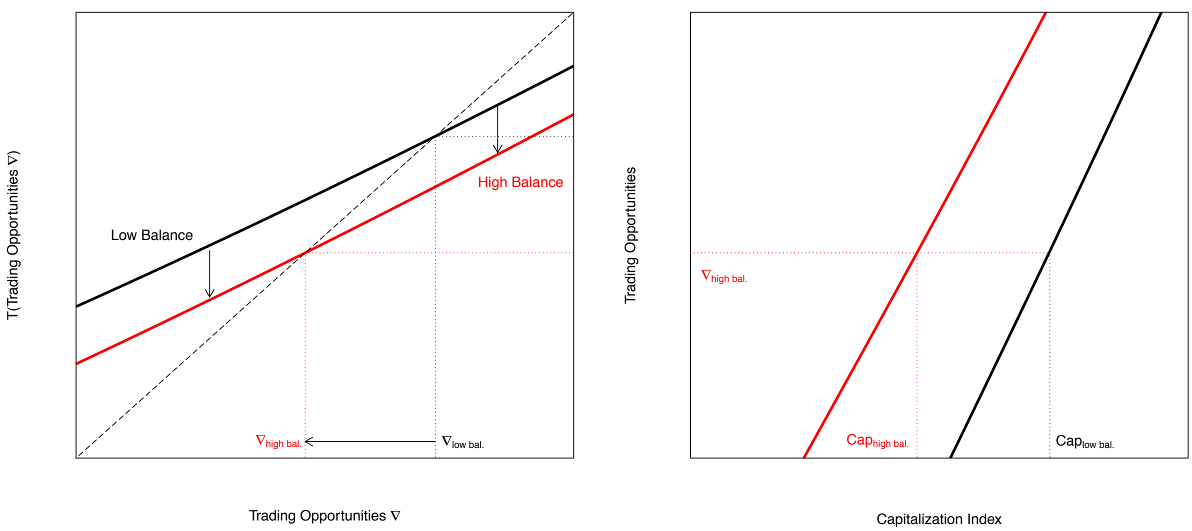
log Mortgage Balance over Price: 1st Quartile 2nd Quartile 3rd Quartile 4th Quartile.
Quartiles taken for the overall 2005-2013 distribution of Trading Opportunities.

Figure 4: Theory Predictions – The Impact of an Increasing Mortgage Indebtedness on the Capitalization of Amenities

This figure presents the impact of a local, neighborhood-level, increase in household indebtedness on trading opportunities and capitalization coefficient. The first impact of a higher indebtedness is a lower trading opportunity index. This is derived from the Bellman equation 7. The second figure shows that a higher indebtedness implies a lower level of capitalization at each level of the trading opportunities index.

(a) Impact on Trading Opportunities

(b) Impact on Capitalization Coefficient



The black line is for the initial balance. The red line corresponds to an increase in the neighborhood's mortgage balance. The dashed line in (a) is the 45 degree line.

Figure 5: The Spatial Distribution of Trading Opportunities

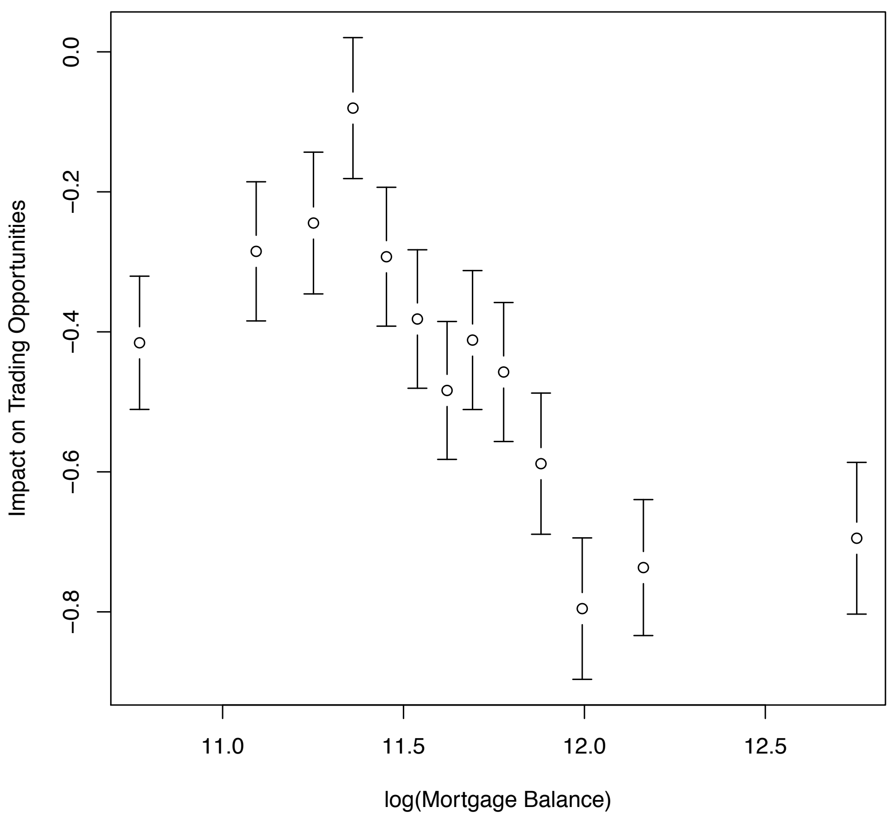
These map focus on Cook county neighborhoods, which includes the City of Chicago, to illustrate the decline of trading opportunities ∇_j *in the southern neighborhoods of Chicago from the peak of the boom in 2007 to the trough of the bust in 2010.*

(a) 2007 (b) 2010

Trading Opportunities: 1st Quartile 2nd Quartile 3rd Quartile 4th Quartile.
Quartiles taken for the overall 2005-2013 distribution of Trading Opportunities.

Figure 6: The Impact of the Increase in Mortgage Indebtedness on Local Trading Opportunities

This figure presents the impact of the rise in local mortgage indebtedness on neighborhood-level trading opportunities. Each point is the coefficient in the regression of trading opportunities on indicator variables for the 20 bins of mortgage indebtedness, where the regression controls for credit conditions, amenities, blockgroup and year fixed effects. The segments are 95% confidence intervals, with standard errors two-way clustered at the neighborhood and year levels.



Mortgage balance: neighborhood-level mortgage balance averaged across all parcels (APNs) of the neighborhood. Data Appendix Section B describes the data construction process that builds a parcel-specific debt level. Trading Opportunities estimated as in Step #1 of the Identification Section 4.

Figure 7: The Price-Amenity Gradient: Preferences vs. Trading Opportunities

This figure reports the coefficients of the regression of the log transaction price in each neighborhood on their current amenity, interacted with an indicator variable for each year, and controlling for blockgroup fixed effects and year fixed effects. Thus the plot coefficients measure the price-amenity relationship each year between 2005 and 2013.

The dotted line is the same set of coefficients, when controlling for the value of trading opportunities, interacted with neighborhood-level credit conditions: balances outstanding, approval rate, foreclosures per housing unit.

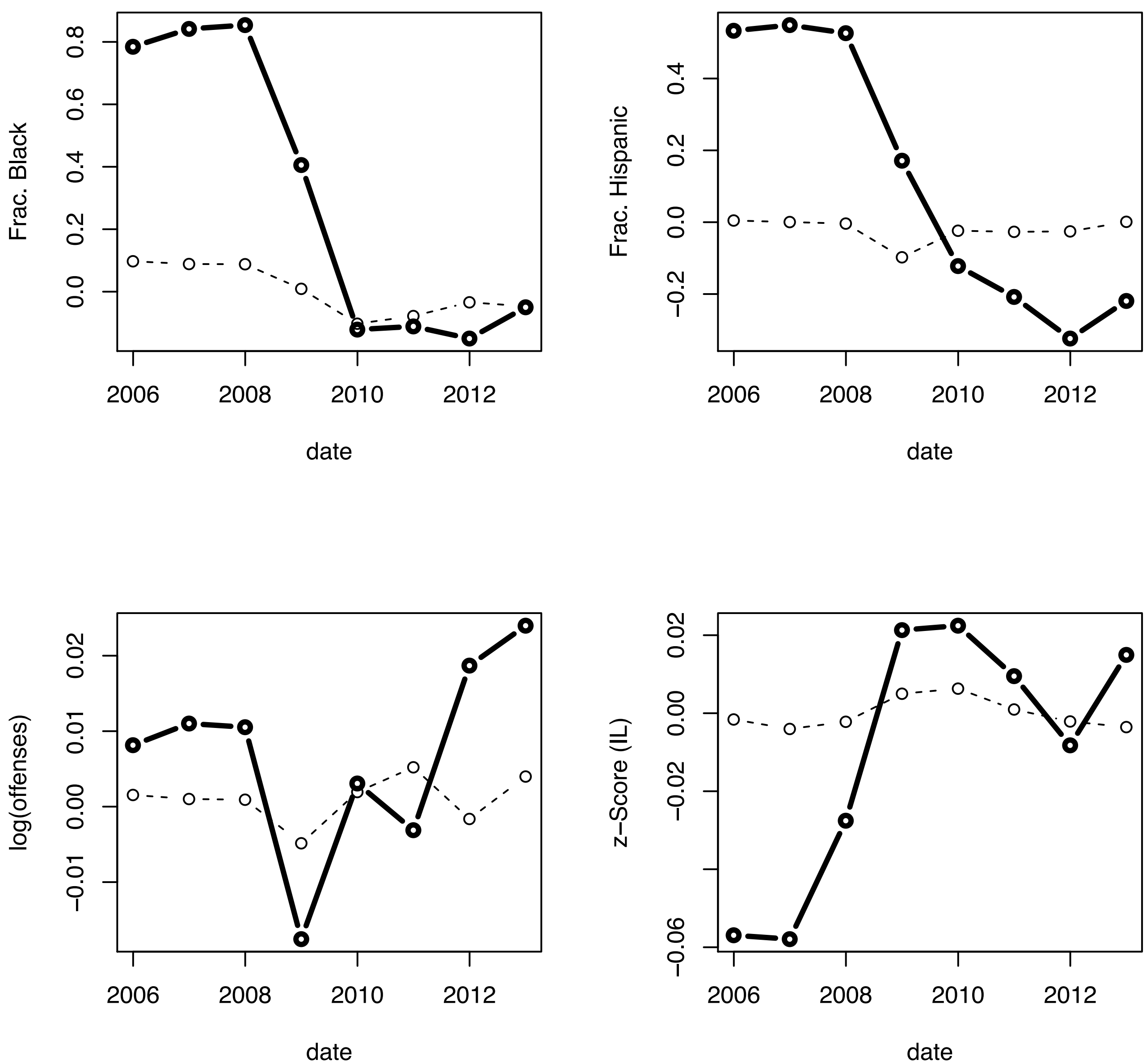
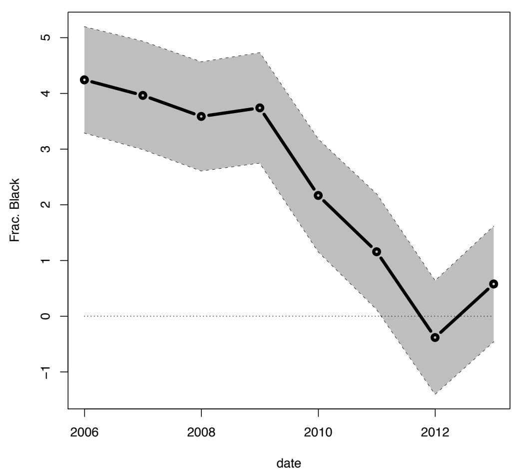


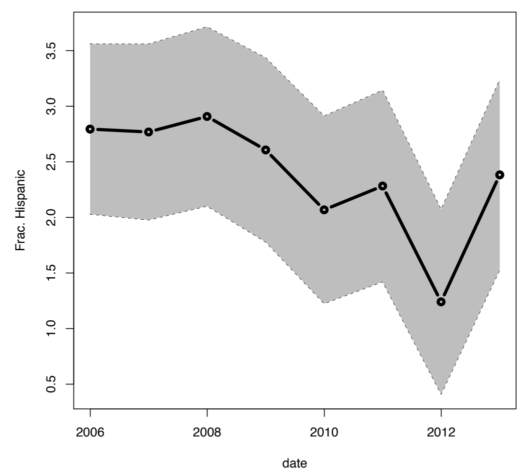
Figure 8: Trading Opportunities and Neighborhood Characteristics

The four figures present scatter plots of the coefficients of the regression of the trading opportunities index ∇ *on a selected group of neighborhood characteristics, interacted with years. Each point is a regression coefficient, grey bands are for 95% confidence intervals.*

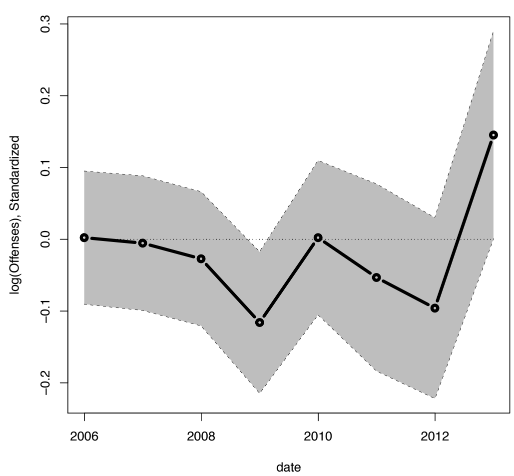
(a) Fraction Black and Trading Opportunities



(b) Fraction Hispanic and Trading Opportunities



(c) Criminal Offenses and Trading Opportunities



(d) School Score and Trading Opportunities

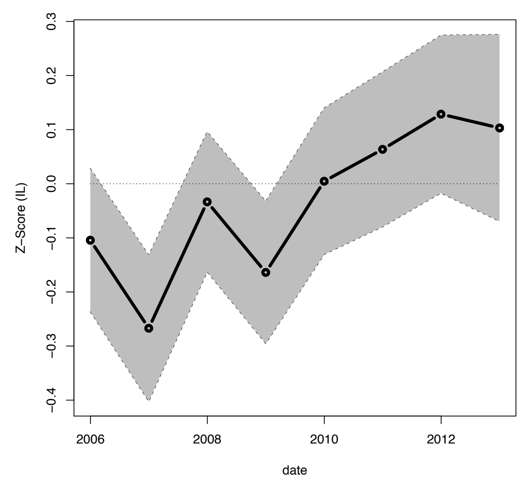


Table 1: Model Estimation – Preferences, Trading Opportunities, and Price-Amenity Gradients When Accounting for Credit Opportunities

This table presents the estimation of preferences $\boldsymbol{\xi}$ *for amenities* $\mathbf{z}_{jt}$ *and the intertemporal discount factor* β *in columns (1) and (3). Columns (2) and (4) present the regression of the trading index* ∇_{ijt}*,* $j = 1, 2, \ldots, J$*, on amenities* $\mathbf{z}_{jt}$*. The trading index* ∇_{ijt} *is the difference between the owner's value* V^{ℓ}_{ijt} *of owning and occupying a unit and the value* V^{v} *of owning a vacant or rented unit. Columns (1) and (2) are estimated in the model with both credit and matching frictions (Section 2.3). Columns (3) and (4) are estimated in the model with matching frictions only (Section 2.1).*

Dependent variable:	(1) Utility V^{ℓ}_{ijt}		(2) Index of Trading Opportunities ∇_{ijt}		(3) Utility V^{ℓ}_{ijt}		(4) Index of Trading Opportunities ∇_{jt}	
	Estimated with Matching and Credit Frictions				Estimated with Matching Frictions Only			
log(Median Income)	0.048	(0.038)	0.026	(0.035)	−0.009	(0.006)	−0.006	(0.016)
Frac. Black	0.320	(0.343)	1.595***	(0.316)	0.036	(0.051)	−0.755***	(0.134)
Frac. Black × Black	9.084***	(0.756)	11.373***	(0.697)	0.818	(1.961)	−8.267	(5.131)
Frac. Hispanic	−0.624**	(0.269)	0.621**	(0.247)	−0.169***	(0.042)	−1.316***	(0.110)
Frac. Hispanic × Hispanic	8.582***	(0.573)	13.086***	(0.526)	0.219**	(0.091)	4.727***	(0.236)
Frac. Asian	−1.281***	(0.417)	−1.538***	(0.384)	−0.049	(0.067)	0.104	(0.174)
Frac. Asian × Asian	14.900***	(0.977)	23.259***	(0.900)	0.019	(0.139)	1.675***	(0.363)
log Offenses	−0.004	(0.009)	−0.043***	(0.008)	−0.002	(0.001)	0.003	(0.004)
Mean Test Score (IL)	−0.059***	(0.022)	−0.096***	(0.020)	0.006	(0.004)	−0.010	(0.009)
Median Test Score (WI)	−1.993	(5.837)	−3.382	(5.376)	−0.417	(0.915)	0.485	(2.394)
Year Structure Built	0.046***	(0.003)	0.041***	(0.003)	0.004***	(0.001)	0.009***	(0.001)
Median Nbr Rooms	0.809***	(0.045)	1.127***	(0.042)	0.102***	(0.007)	0.081***	(0.019)
$E(V^l(j', t+1) \mid j, t)$	0.583***	(0.005)	-		0.518***	(0.001)	-	
Observations	118,053†		118,053†		90,865†		90,865†	
R^2	0.842		0.845		0.997		0.993	

*p<0.1; **p<0.05; ***p<0.01

†: *These specifications have one observation by demographic group* i*, neighborhood* j*, and year* t*.*

Table 2: The Reversal of the Correlation Between Prices and School Test Scores

This table presents the time varying coefficient of the regression of transaction prices (column (1)), trading opportunities (column (2)), and expectations (column (3)) on standardized school test scores.

	Dependent variable:					
	log(Transaction Price$_{jt}$)		Trading Opportunities ∇_{jt}		$\mathbb{E}(V_{jt}^{\ell})$	
	(1)		(2)		(3)	
log(Median Household Income)	0.041^{**}	(0.019)	0.017	(0.036)	0.012^{***}	(0.002)
Frac Black	−0.120	(0.080)	0.475^{***}	(0.182)	-0.040^{***}	(0.006)
Frac Hispanic	-0.166^{***}	(0.057)	0.446^{***}	(0.126)	-0.012^{***}	(0.004)
Frac Asian	−0.069	(0.088)	0.174	(0.217)	0.109^{***}	(0.008)
Average Test Score (IL) × 2005	-0.156^{***}	(0.027)	-0.096^{***}	(0.035)	-0.025^{***}	(0.001)
Average Test Score (IL) × 2006	-0.175^{***}	(0.027)	-0.096^{***}	(0.035)	-0.024^{***}	(0.001)
Average Test Score (IL) × 2007	-0.143^{***}	(0.027)	-0.073^{**}	(0.035)	-0.023^{***}	(0.001)
Average Test Score (IL) × 2008	0.025	(0.028)	−0.056	(0.034)	-0.018^{***}	(0.001)
Average Test Score (IL) × 2009	0.128^{***}	(0.027)	0.045	(0.034)	-0.015^{***}	(0.001)
Average Test Score (IL) × 2010	0.114^{***}	(0.027)	0.053	(0.035)	-0.008^{***}	(0.001)
Average Test Score (IL) × 2011	0.081^{***}	(0.026)	0.047	(0.035)	-0.015^{***}	(0.001)
Average Test Score (IL) × 2012	0.077^{***}	(0.027)	0.043	(0.038)	−0.0001	(0.002)
Share of Schools with B	0.018	(0.061)	−0.084	(0.131)	0.009^{**}	(0.004)
Share of Schools with C	-0.205^{***}	(0.053)	0.026	(0.111)	0.019^{***}	(0.003)
Share of Schools with D	-0.111^{*}	(0.061)	0.107	(0.133)	-0.021^{***}	(0.003)
Share of Schools with F	−0.104	(0.145)	0.291	(0.208)	-0.092^{***}	(0.009)
Median Test Score (WI)	−0.045	(1.220)	0.195	(0.890)	0.158^{***}	(0.054)
Median Year Structure Built	0.004^{***}	(0.001)	0.007^{***}	(0.002)	0.0001	(0.0001)
Median Number of Rooms	0.032^{***}	(0.010)	0.176^{***}	(0.023)	0.004^{***}	(0.001)
log(Offenses Per Capita)	−0.376	(0.686)	−1.318	(0.986)	-0.228^{***}	(0.045)
Observations	37,564		37,666		37,666	
R^2	0.793		0.822		0.99	
Adjusted R^2	0.754		0.789		0.99	
Residual Std. Error	0.358 (df = 31574)		0.461 (df = 31673)		0.020 (df = 31673)	
Note:					*p<0.1; **p<0.05; ***p<0.01	

Table 3: The Capitalization of Trading Opportunities Into House Prices

This regression tests the predictions of Section 3 (The Price-Amenity Gradient at Equilibrium). The second column estimates the impact of trading opportunities on observed transaction prices. The third column estimates how credit frictions affects such capitalization of trading opportunities into house prices.

	Dependent variable:					
	log(Transaction Price$_{jt}$)					
	(1)		(2)		(3)	
log(Median Household Income)	0.044**	(0.022)	0.001	(0.019)	0.005	(0.015)
Frac. Black	−0.160*	(0.090)	−0.118	(0.085)	−0.106*	(0.057)
Frac. Hispanic	−0.201***	(0.060)	−0.230***	(0.058)	−0.081*	(0.042)
Frac. Asian	0.032	(0.090)	−0.363***	(0.095)	−0.180***	(0.066)
Mean Test Score (IL)	−0.030***	(0.006)	0.016***	(0.006)	0.004	(0.005)
Share of Schools with B (IN)	0.015	(0.060)	0.004	(0.063)	0.006	(0.051)
Share of Schools with C (IN)	−0.201***	(0.053)	−0.265***	(0.058)	−0.161***	(0.045)
Share of Schools with D (IN)	−0.108*	(0.061)	−0.061	(0.067)	−0.056	(0.055)
Share of Schools with F (IN)	−0.093	(0.144)	0.141	(0.133)	−0.039	(0.124)
Median Test Score (WI)	−0.027	(1.219)	−0.557	(1.239)	−0.175	(1.047)
Median Year Structure Built	0.004***	(0.001)	0.002***	(0.001)	0.001	(0.001)
Median Number of Rooms	0.050***	(0.012)	0.002	(0.011)	0.011	(0.008)
Trading Opportunities ∇			0.169***	(0.008)	0.032***	(0.005)
$\times$ log Balance					−0.003***	(0.001)
$\times$ Approval Rates					0.352***	(0.042)
$\times$ log(Interest Cost)					0.005	(0.003)
$\mathbb{E}(V^{\ell})$			3.090***	(0.132)	1.281***	(0.106)
log(Offenses per Capita)	−5.474***	(0.695)	−3.338***	(0.664)	−2.329***	(0.549)
Other Controls	log(Balance), Approval Rate, log(Interest Cost)					
Observations	37,564		37,564		37,564	
R^2	0.772		0.788		0.844	
Adjusted R^2	0.728		0.747		0.814	
Residual Std. Error	0.376 (df = 31581)		0.363 (df = 31579)		0.311 (df = 31573)	
Note:	*p<0.1; **p<0.05; ***p<0.01					

The index ∇ of trading opportunities is standardized, s.d.$(\nabla) = 1$ across neighborhoods.

A City Dynamics with Credit Constraints

This section of the Appendix presents the equations describing the city's dynamics with credit frictions. The city's equilibrium is characterized by a vector of neighborhood-level trading opportunities $\boldsymbol{\nabla}$, a vector of listings $\boldsymbol{\ell}$, and a vector of mortgage balances $\mathbf{B}$. The dynamics of trading opportunities with credit frictions are affected by the probability of approval $\varphi(j, \mathbf{Z})$ in each neighborhood j for a given distribution of city-wide amenities; these approval probabilities lower the probability of a successful transaction. The dynamics are also affected by the mortgage balance in buyers' neighborhoods B and the interest cost at destination. Overall, the Bellman equation for trading opportunities ∇ is:

$$\begin{aligned}\nabla(j,\mathbf{Z}) = \mathbf{z}_j(\boldsymbol{\xi}-\boldsymbol{\zeta}) + \beta\mathbb{E}\nabla(j,\mathbf{Z}') \\ + \beta\mathbb{E}\Bigg[\sum_{j'=1}^{J}\big\{(1-\gamma)\delta\lambda(j',\mathbf{Z}')\varphi(j',\mathbf{Z}')\boldsymbol{\Phi}(\nabla(j',\mathbf{Z}') - \kappa(j',\mathbf{Z}') - \nabla(j,\mathbf{Z}') - B(j,\mathbf{Z}')) \\ + \gamma\delta^v\mu(j,\mathbf{Z}')\varphi(j,\mathbf{Z})\boldsymbol{\Phi}(\nabla(j,\mathbf{Z}') - \kappa(j,\mathbf{Z}') - \nabla(j',\mathbf{Z}') - B(j',\mathbf{Z}'))\big\}\Bigg] + \varepsilon.\end{aligned} \tag{29}$$

The dynamic of listings is affected as well by the presence of credit frictions. Approval rates, mortgage balances, and interest payments lower both flows into and out of a neighborhood.

$$\begin{aligned}\Delta\ell(j,\mathbf{Z}) = \Bigg\{\delta n(j,\mathbf{Z})\sum_{j'=1}^{J}\lambda(j',\mathbf{Z})\varphi(j,\mathbf{Z})F\left[\nabla(j,\mathbf{Z}) - \kappa(j,\mathbf{Z}) - \nabla(j',\mathbf{Z}) - B(j',\mathbf{Z})\right] \\ - \delta^v\ell(j,\mathbf{Z})\sum_{j'=1}^{J}\mu(j',\mathbf{Z})\varphi(j',\mathbf{Z})F\left[\nabla(j',\mathbf{Z}) - \kappa(j',\mathbf{Z}) - \nabla(j,\mathbf{Z}) - B(j,\mathbf{Z})\right]\Bigg\},\end{aligned} \tag{30}$$

The third dynamics is the dynamics of mortgage balances, presented in equation (13) in the main body of the paper.

B Data Appendix

The paper combines five data sets to estimate the structural parameters. This section describes how these data sets are built to estimate price-amenity gradients and households' preferences for amenities.

Deeds records come from **Corelogic Inc.**, which compiled information from the 14 county recording offices of the Chicago metropolitan area between 2000 and 2014 inclusive. The latitude and longitude of each parcel was obtained by geocoding the street address using the Census PostGIS TIGER Geocoder for the three states of the MSA (Illinois, Indiana, Wisconsin). While street addresses are encoded as text, the success rate for matches was above 80%. Each transaction's latitude and longitude is then matched to each of the Census 2010 blockgroup boundaries. The characteristics of the mortgage are added to the transaction as follows. When a grant deed was recorded, the characteristics of the mortgage are typically observed in the same record as the property transaction. When a deed of trust was recorded, the characteristics of the mortgage are recorded separately from the property transaction. In this case, we build a parcel specific timeline to associate the mortgage characteristics with the last property transaction on the same parcel.

The race, ethnicity, and income of the buyer are obtained by matching the deeds records to **data collected in accordance with the Home Mortgage Disclosure Act**. This is done by matching each of the transactions by loan amount and census tract. As Home Mortgage Disclosure Act data uses 1990 boundaries until 2003 inclusive and 2000 boundaries after 2004 inclusive, we match deeds to HMDA using HMDA's boundaries, and then merge them to consistent 2010 Census tract boundaries.

Blockgroup-level school quality measures are obtained by using the polygon boundaries of school attendance areas, the geocoding of schools by the federal **National Center for Education Statistics**, and school-level test scores from each of the three states. School attendance boundaries are obtained by hand coding the polygons of **Great Schools**. The geographic location of each school is included in the National Center for Education Statistics' **Common Core of Data**. Thus schools are matched to their corresponding attendance area. Each school's state-specific test score is obtained by matching the school's state identifier with the state's test score data. This is possible as the Common Core of Data includes both the federal identifier and the state-level identifier. Test scores are standardized at the year and state-level. Each property is associate to a test score in the following way. First, blockgroup boundaries are intersected with school attendance area boundaries. Second, if a blockgroup lies entirely within a school boundary, the test score for this property is the test score of the attendance area. Third, if a blockgroup boundary crosses two or multiple school attendance areas, the test score is the weighted average of the test scores of the corresponding attendance areas, weighted by the area of their intersections.

Crime data comes from the **FBI's Uniform Crime Reports** and from the **City of Chicago's Crime Data Portal**. The Uniform Crime Reports report crime counts by Originating Agency Identifier. The FBI provides a crosswalk from ORIs to incorporated Census places. The City of Chicago's data reports geocoded criminal incidents. In the latter case, such events are matched to blockgroup boundaries within the city of Chicago. In the former case, the counts are apportioned to blockgroups in the following way. First, special police data (e.g. airport police or university police) is eliminated. Second, blockgroups are intersected with the boundaries of Census places. When a blockgroup does not intersect with a Census place, the crime count per resident is that of the County police. When the blockgroup is within a Census place, the crime count is that of the municipal police of the place. Some police forces serve multiple incorporated municipalities.

Neighborhood demographics come from the **American Community Survey**. As blockgroup boundaries change with every decennial Census and thus also for waves of the ACS between 2005 and 2014, we build a consistent longitudinal data set of neighborhoods with 2010 blockgroup boundaries in the following way. We build a blockgroup relationship file by intersecting the boundaries of the 2000 Census Shapefiles with the boundaries of the 2010 Census Shapefiles. Counts with 2000 boundaries are apportioned according to the size of the intersection. Averages are the area-weighted averages of the intersections. Since the ACS provides 5-year averages, we build annual values by observing that, for a Census variable x_{jt} (unobserved) for each neighborhood j and year t, the ACS reports: $\overline{x}_{jy_0} = \frac{1}{5}(x_{jy_0} + x_{jy_0+1} + x_{jy_0+2} + x_{jy_0+3} + x_{y_0+4})$ for $y_0 = 2005, \ldots, 2010$. Thus the identification problem is to infer 10 data points x_{jy} for $y = 2005, 2014$ from the observation of 6 per neighborhood. A polynomial regression of $\overline{x}_{jy}$ on t for each data set identifies each of the 6003 blockgroups' trend in $\overline{x}_{jy}$. Then the relationship between the polynomial coefficients of $\overline{x}_{jy}$ and x_{jy} provides the polynomial coefficients of x_{jy} and the imputed annual value for annual demographics.

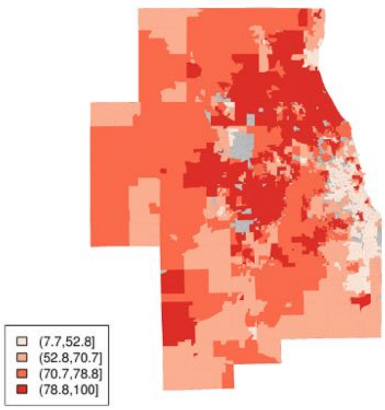
C Estimating Lending Standards: The Self-Selection Problem

The availability of the HMDA microfiles enables the estimation of a lending standards specification where the approval decision (approved/denied) depends on applicant, mortgage, and house char-

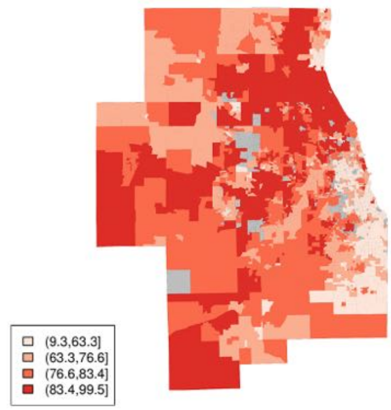
Appendix Figure A: Assigning School Test Scores to Neighborhoods in the Illinois neighborhoods of the Chicago-Naperville-Elgin Metro Area

School-level test scores from Illinois' State Board of Education are matched to their corresponding school attendance area. Each neighborhood is then assigned the test score from its corresponding attendance area. Darker colors indicate higher test scores.

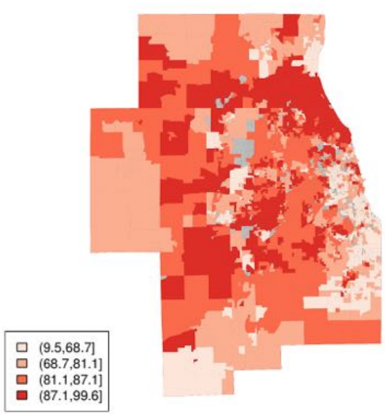
(a) 2005



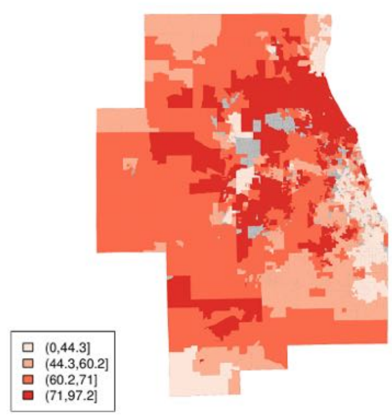
(b) 2006



(c) 2012

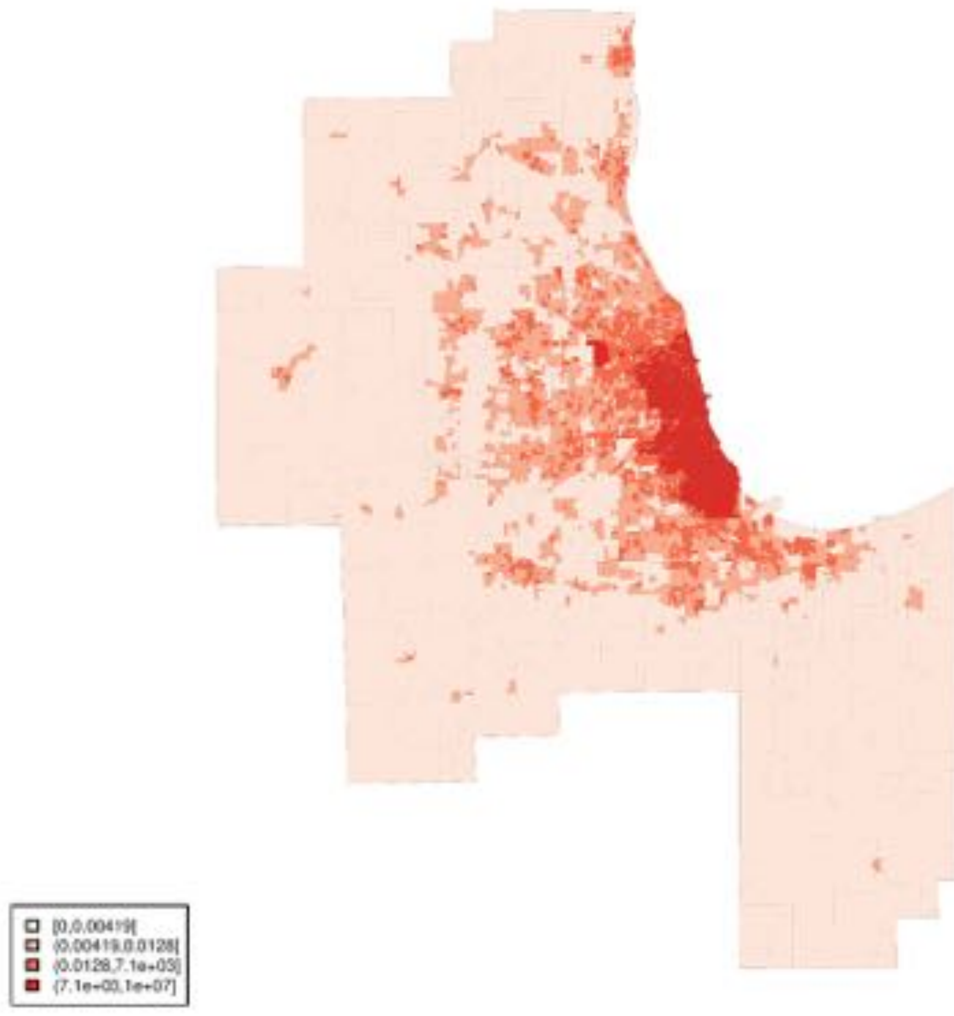


(d) 2013



Appendix Figure B: Crime Rates by Blockgroup

This map shows the counts of crimes per 100,000 residents obtained for each blockgroup of the Chicago-Naperville-Elgin, IL-IN-WI Metropolitan Statistical Area using the approach described in Appendix Section B. Data comes from the City of Chicago's Data portal for crimes committed within the boundaries of the City of Chicago. For neighborhoods outside of the city, Uniform Crime Report (UCR) data was used.



acteristics. Modelling this approval decision as the lender's benefit-cost analysis of lending, the probability of approval φ_{ijt} for household i in neighborhood j in year t is:

$$\varphi_{ijt} = P(\mathbf{x}_{it}\boldsymbol{\beta}_t + \mathbf{z}_{jt}\boldsymbol{\gamma}_t + \mathbf{x}_{it}\boldsymbol{\Psi}_t\mathbf{z}_{jt} + \varepsilon_{ijt} > 0) \tag{31}$$

where $\mathbf{x}_{it}$ is the vector of household characteristics, $\mathbf{z}_{jt}$ is the vector of mortgage and house characteristics. ε_{ijt} is the unobservable driver of the approval decision. The evolution of lending standards is then the evolution of the estimates of the coefficients $\boldsymbol{\beta}_t$, $\boldsymbol{\gamma}_t$, and $\boldsymbol{\Psi}_t$ between $t = 2005$ and $t = 2013$.

One significant empirical issue in the estimation is that the volume of mortgage applications drops dramatically from the market's boom to the market's bust: going from above 300,000 applications to less than 100,000. Thus households' self-selection causes large shifts in the sample of estimation, which confounds the estimation of these lending standards coefficients.

We address this issue as follows. The distribution of characteristics $(\mathbf{x}, \mathbf{z})$ in year t is noted $f(\mathbf{x}, \mathbf{z}|t)$. We reweigh the sample so that the distribution $f(\mathbf{x}, \mathbf{z}|t)$ is identical regardless of the value of t. A practical issue is that $f(\mathbf{x}, \mathbf{z}|t)$ is a non-parametric statistical object. Our method involves the following simple steps. By Bayes' rule:

$$f(\mathbf{x}, \mathbf{z}|t) = \frac{f(t|\mathbf{x}, \mathbf{z})f(\mathbf{x}, \mathbf{z})}{f(t)} \tag{32}$$

where $f(t)$ is the share of observations in year t. The probability $f(t|\mathbf{x}, \mathbf{z})$ that an observation $(\mathbf{x}, \mathbf{z})$ is in year t is estimated as the prediction of the linear regression of an indicator variable for year t on the set of covariates $\mathbf{x}$ and $\mathbf{z}$. The set of weights for year t is thus:

$$w(\mathbf{x}, \mathbf{z}, t) = \frac{f(2005|\mathbf{x}, \mathbf{z})}{f(2005)} \cdot \frac{f(t)}{f(t|\mathbf{x}, \mathbf{z})} \tag{33}$$

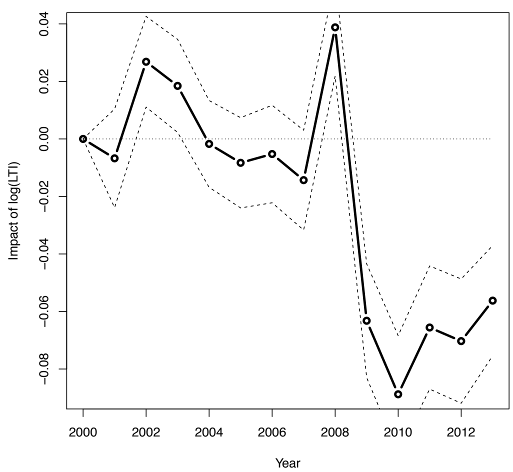
which ensures that any distribution for $t = 2006, \ldots, 2013$ is identical to the distribution of observations in $t = 2005$. For instance the weighted mean of $\mathbf{x}$ for any $t \geq 2006$ is

$$\int w(\mathbf{x}, \mathbf{z}, t) \cdot \mathbf{x} \cdot f(\mathbf{x}, \mathbf{z}|t) d\mathbf{x}d\mathbf{z} = \int \mathbf{x} \cdot \frac{f(2005|\mathbf{x}, \mathbf{z})}{f(2005)} f(\mathbf{x}, \mathbf{z}) d\mathbf{x}d\mathbf{z} = \int \mathbf{x} \cdot f(\mathbf{x}, \mathbf{z}|2005) d\mathbf{x}d\mathbf{z} \tag{34}$$

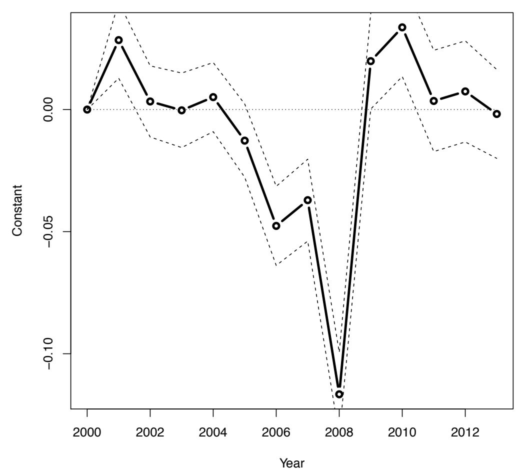
Appendix Figure C: Estimation of the Evolution of Lending Standards: Approval Rate

The figures below present the evolution of credit frictions. Specifically, figure (a) presents the estimated sensitivity of the mortgage approval decision to the applicant's loan-to-income ratio. Figure (b) presents the constant of the approval specification. Figure (c) presents the

(a) Approval Specification – Sensitivity to the LTI



(b) Approval Specification – Constant



which is independent of t. This is true for any moment of the observations. This approach enables the estimation of lending standards without shifts in the distribution of observations across years. This method is used in Appendix Figure C.

D Theory

D.1 Existence and Uniqueness of the Steady-State Equilibrium

Proposition 6. (Existence and Uniqueness of a Steady-State City Equilibrium)

Proof. We consider the space noted $\mathcal{S}$ of couples (∇, ℓ) of a trading opportunities function of amenities $\nabla : \{1, 2, \ldots, J\} \times \mathbb{R}^{JK} \to \mathbb{R}$, $(j, \mathbf{Z}) \mapsto \nabla(j, \mathbf{Z})$ and a listing function $\ell : \{1, 2, \ldots, J\} \times \mathbb{R}^{JK} \to \mathbb{R}$, $(j, \mathbf{Z}) \mapsto \ell(j, \mathbf{Z})$.

First, amenities $\mathbf{z}$ belong to a closed and bounded, i.e. compact set $\mathcal{Z}$ of $\mathbb{R}^K$. This assumption has two implications: first, the city-wide distribution of amenities $\mathbf{Z}$ also belongs to a compact set as the cartesian product of compacts. Second, values V^l and V^v are bounded. Therefore ∇ is a function mapping from a compact set into a compact set $\mathcal{D} \subset \mathbb{R}$. Similarly, listings $l(j, \mathbf{Z})$ evolve in a compact set $[0, h_j]$ for each j and for all $\mathbf{Z} \subset \mathcal{Z}$.

Second, the index of trading opportunities ∇ satisfies equation 7. This is expressed in terms of operator T_∇ from (∇, ℓ) to ∇, i.e. $\nabla = T_\nabla(\nabla, \ell)$ in steady-state. Similarly, the dynamics of listings can be expressed as an operator T_ℓ from (∇, ℓ) to ℓ, i.e. $\ell = T_\ell(\nabla, \ell)$ in steady-state. The steady-state is a couple (∇^*, ℓ^*) such that $(\nabla^*, \ell^*) = T(\nabla^*, \ell^*)$, with $T = (T_\nabla, T_\ell)$.

Finally, the vector space $\mathcal{S}$ is endowed with a norm $\|\cdot\|_\infty$ with $\|(\nabla, \ell)\|_\infty = \max\{|\nabla|, |\ell|\}$. Hence $\mathcal{S}$ is a Banach space. The mapping T is a Lipschitz contraction from $\mathcal{S}$ into itself with constant $\max\{\beta, \delta, \delta^v\} < 1$. By the Banach–Caccioppoli fixed-point theorem there is a unique fixed point $(\nabla^*, \ell^*) \in \mathcal{S}$ of such mapping. □

D.1.1 Value Function Iterations

The comparative statics that provide the predictions of Section (E.1) are performed as follows. Keeping the preferences of households $\boldsymbol{\xi}$ constant, the steady-state[5] neighborhood-level trading opportunities index ∇ and the neighborhood-level listings ℓ satisfy 7 and 8 for a given value of city-wide market tension L/H, city-wide listings divided by the number of housing units. In steady-state, listings are stable, $\Delta\ell = 0$. The index of trading opportunities (resp. the listings function) is a function ∇ of the vector of amenities $\mathbf{z} \in \mathbb{R}^K$ (resp., a function ℓ of $\mathbf{z}$). The space of amenities is bounded, and discretized on an equally-spaced grid. We use value function iteration over $\nabla(\mathbf{z})$ and $\ell(\mathbf{z})$ to obtain

[5] The model also allows non-steady-state dynamics with a path of shifts in market tension.

the corresponding steady-state functions, and Chebyshev polynomial approximations (Judd 1998) to estimate values of $\nabla(\mathbf{z})$ and $\ell(\mathbf{z})$ for amenities outside of the grid.

As the values of ownership and vacancies V^l and V^v are simple functions (3) and (4) of the index of trading opportunities and listings, values and prices $\log p(\mathbf{z}', \mathbf{z})$ for each origin-destination neighborhood pair $(\mathbf{z}', \mathbf{z})$ are straightforwardly derived from the simulation of ∇ and ℓ. Average prices $E_{\mathbf{z}'}(\log p(\mathbf{z}', \mathbf{z}))$ in neighborhood with amenities $\mathbf{z}$ are obtained by averaging $\log p(\mathbf{z}', \mathbf{z})$ over the probabilities of transaction between $\mathbf{z}'$ and $\mathbf{z}$. The price-amenity gradient is the simple regression of such average transaction price $E_{\mathbf{z}'}(\log p(\mathbf{z}', \mathbf{z}))$ on the amenity vector $\mathbf{z}$.

D.2 Short-Run Transition Dynamics

At the city's steady-state, the amenity-value difference ∇ and listings ℓ are constant for a given distribution of city-wide amenities $\mathbf{Z}$. How the city converges to such steady-state remains to be established: shocks to preferences $d\boldsymbol{\xi}$, shocks to amenities $d\mathbf{z}_j$ in a neighborhood, and shocks to the matching technology $d\delta$, $d\delta^v$, are incorporated into values with some delay. Deriving the short run dynamics of (∇, ℓ) will give an understanding of the short-run dynamics of prices and transaction volumes; in particular, it will show with what speed shifts in the value of amenities are transmitted into house prices.

We consider a permanent shift in a structural parameter $\boldsymbol{\theta}$, a notation that encompasses the three specific cases of preferences, amenities, and the matching technology. The parameter θ evolves according to a discrete-time sequence (θ_t) where $\theta_0 = \underline{\theta}$ in the initial period, and $\theta_t = \overline{\theta}$ for all time periods $t \geq \bar{t}$. The values and listings in period $t = 0, 1, 2, \ldots$ are noted (∇_t, ℓ_t).

Proposition 7. (Short-Run City Dynamics in Response to a Shock) *Given an anticipated path for a structural parameter θ according to a discrete-time sequence (θ_t), and given an initial condition (∇_0, ℓ_0) there exists a unique sequence (∇_t, ℓ_t) such that households' amenity-value gap are pinned down according to Bellman dynamics (7), where the current ∇_t depends on forward-looking ∇_{t+1} and ℓ_t, ℓ_{t+1}; and where listings ℓ_t evolve according to (8).*

As $t \to \infty$, (∇_t, ℓ_t) converges[6] to the steady-state $(\nabla_\infty, \ell_\infty)$ that would be achieved if θ were constant and equal to $\lim_{t\to\infty} \theta_t$ in all time periods. Similarly the amenity-price gradient converges to the steady-state value $(\partial \log \mathbf{p}/\partial \mathbf{z})_\infty$.

[6] The norm $\|\cdot\|$ that defines the convergence concept is defined in the Appendix.

Proof. The Bellman dynamics (7) and (8) in the out-of-steady state path can be expressed as two operators:

$$\begin{cases} \nabla_t & = & T_\nabla(\nabla_{t+1}, \ell_{t+1}) \\ \ell_{t+1} & = & T_\ell(\nabla_t, \ell_t) \end{cases} \tag{35}$$

where $\nabla_t \to \nabla_\infty$ and $l_t \to l_\infty$ as $t \to \infty$. The operator T_∇^k is the k times composition so that the amenity-price gap can be expressed as a function $\nabla_t = T_\nabla^k(\nabla_{t+k}, \ell_t)$ of the forward looking $t+k$ value of ∇. Then, when $\max\{\beta, \delta, \delta^v\} < 1$, as $t \to \infty$, the operator T_∇^k converges to an operator T_∇^∞ and the amenity-price gap converges to a value ∇_∞. Thus the current value can be simply expressed as the steady-state value and the current listings $\nabla_t = T_\nabla^\infty(\nabla_\infty, \ell_t)$. Indeed, the convergence point ∇_∞ is well defined according to proposition (1).

The dynamics become:

$$\begin{cases} \nabla_t & = & T_\nabla^\infty(\nabla_\infty, \ell_t) \\ \ell_{t+1} & = & T_\ell(\nabla_t, \ell_t) \end{cases} \tag{36}$$

which leads to the unique transition sequence (∇_t^*, ℓ_t^*), for $t = 1, 2, \ldots$. □

The proof of Proposition 7 shows that from a given initial condition (a vector of listings per neighborhood $\boldsymbol{\ell} \in \mathbb{R}^J$) and from a path of parameters, e.g. shifts in amenities, there is a unique path for city dynamics.

D.3 Closed-Form Expression of the Capitalization Coefficient

Proposition 8. (Impact of trading opportunities on price-amenity gradient)

Proof. In the price-amenity gradient $\partial \log \mathbf{p}_{kj}/\partial \mathbf{z}_j$, collect the terms in $\partial\nabla(j, \mathbf{Z})/\partial \mathbf{z}_j$:

$$\begin{aligned} \frac{\partial \log \mathbf{p}_{kj}}{\partial \mathbf{z}_j} &= \gamma \frac{\partial V^l(j, \mathbf{Z})}{\partial \mathbf{z}_j} + (1-\gamma)\frac{\partial V^v(j, \mathbf{Z})}{\partial \mathbf{z}_j} \\ &= \gamma \boldsymbol{\xi} + (1-\gamma)\boldsymbol{\zeta} + \gamma\beta \frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^l(j, \mathbf{Z}')|\mathbf{Z}\right] + (1-\gamma)\beta\frac{\partial \mathbb{E}}{\partial \mathbf{z}_j}\left[V^v(l, \mathbf{Z}')|\mathbf{Z}\right] \\ &\quad - \gamma(1-\gamma)\beta\delta \sum_{j'=1}^{J} \lambda_{j'} \Phi'(\nabla(j', \mathbf{Z}) - \nabla(j, \mathbf{Z}))\frac{\partial \nabla(j, \mathbf{Z})}{\partial \mathbf{z}_j} \\ &\quad + \gamma(1-\gamma)\beta\delta^v \sum_{j'=1}^{J} \mu_{j'} \Phi'(\nabla(j, \mathbf{Z}) - \nabla(j', \mathbf{Z}))\frac{\partial \nabla(j, \mathbf{Z})}{\partial \mathbf{z}_j}, \end{aligned}$$

and the factor in $\gamma(1-\gamma)\beta\partial\nabla(j,\mathbf{Z})/\partial\mathbf{z}_j$ (last two lines) is:

$$\sum_{j'=1}^{J}\left[\delta^v\mu_{j'}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-\delta\lambda_{j'}\Phi'(\nabla(j',\mathbf{Z})-\nabla(j,\mathbf{Z}))\right], \tag{37}$$

where $\Phi'(d)=(\frac{1}{\sqrt{2}\sigma_\varepsilon}-1)f(\frac{d}{\sqrt{2}\sigma_\varepsilon})d+F(\frac{d}{\sqrt{2}\sigma_\varepsilon})$, hence $\Phi'(-d)=1-\Phi'(d)$. Factoring by δ, the term becomes:

$$\begin{aligned}
&\sum_{j'=1}^{J}\left[\frac{N}{L}\mu_{j'}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-\lambda_{j'}(1-\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z})))\right]\\
=&\sum_{j'=1}^{J}\left[\frac{N}{L}\mu_{j'}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-\lambda_{j'}(1-\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z})))\right]\\
=&\sum_{j'=1}^{J}(\frac{N}{L}\frac{n_{j'}}{N}+\frac{\ell_{j'}}{L})\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-1\\
=&\frac{H}{L}\sum_{j'=1}^{J}\frac{h_{j'}}{H}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-1.
\end{aligned}$$

As $\Phi'(\cdot)$ is an increasing function, and $H>L$, there is a value $\underline{\nabla}$ such that $\frac{H}{L}\sum_{j'=1}^{J}\frac{h_{j'}}{H}\Phi'(\underline{\nabla}-\nabla(j',\mathbf{Z}))=1$. At that point, $\partial\log\mathbf{p}_{kj}/\partial\mathbf{z}_j=\gamma\boldsymbol{\xi}$.

Now on average across locations, weighted by their number of housing units h_j/H,

$$\begin{aligned}
&\frac{H}{L}\sum_{j=1}^{J}\sum_{j'=1}^{J}\frac{h_j}{H}\frac{h_{j'}}{H}\Phi'(\nabla(j,\mathbf{Z})-\nabla(j',\mathbf{Z}))-1\\
=&\frac{H}{L}\sum_{j=1}^{J}\sum_{j'=1}^{J}\frac{h_j}{H}\frac{h_{j'}}{H}(\mathbf{1}(j\neq j')+\frac{1}{2}\mathbf{1}(j=j'))-1\\
=&\frac{H}{L}(\sum_{j=1}^{J}\frac{h_j}{H}(1-\frac{h_j}{H})+\frac{1}{2}\sum_{j=1}^{J}\left(\frac{h_j}{H}\right)^2)-1=\frac{H}{L}(1-\frac{1}{2}\sum_{j=1}^{J}\left(\frac{h_j}{H}\right)^2)-1
\end{aligned}$$

the first term is positive when $H>2L$. This makes the log price amenity relationship steeper if $\partial\nabla(j,\mathbf{Z})/\partial\mathbf{z}_j\geq 0$ and flatter if $\partial\nabla(j,\mathbf{Z})/\partial\mathbf{z}_j\leq 0$. If all housing units are concentrated in one neighborhood, the term becomes -1. $\square$

D.4 The Simple Case of a City without Matching Frictions

We consider a set of J neighborhoods and N households indexed by i. The price of housing in neighborhood j is noted p_j.

The demand for housing The value of living in a unit for household i is noted:

$$V^{\ell}(\mathbf{z}) = \mathbf{z}\boldsymbol{\gamma} - p(\mathbf{z}) + \varepsilon + \beta \cdot E(V^{\ell}(\mathbf{z}')|\mathbf{z}) \tag{38}$$

where $p(\mathbf{z})$ is the price of housing in a neighborhood with amenities $\mathbf{z}$. The unobservable ε generates variations in the valuations of housing across households, and thus a smooth downward sloping demand curve. The demand for housing is the probability that such $V^{\ell}(\mathbf{z})$ is maximum across all values of $\mathbf{z}$ on the support $\mathcal{Z}$. The demand for a neighborhood with characteristic $\mathbf{z}$ given the citywide distribution of characteristics $f(\mathbf{z})$ is thus:

$$D(\mathbf{z}; f()) = N \cdot \frac{\exp(V^{\ell}(\mathbf{z}))}{\int_{\mathcal{Z}} \exp(V^{\ell}(w)) f(w) dw}$$

In steady-state, $f(\cdot)$ will be stable over time.

The supply of housing The supply of housing is the number of housing units "for sale" in each neighborhood. This number of units is immediately equal to the number of housing units minus the number of units demanded in the neighborhood. *Demand generates its own supply.*

$$S(\mathbf{z}) = h(\mathbf{z}) - n(\mathbf{z})$$

The value of a vacancy is then noted:

$$V^{v}(\mathbf{z}) = \mathbf{z}\boldsymbol{\xi} + p(\mathbf{z}) + \beta \cdot E(V^{v}(\mathbf{z}')|\mathbf{z})$$

where the owner of a vacant unit earns the market price $p(\mathbf{z})$. This value in turn affects the right-hand side $E()$ of equation (38), and implies that V^{ℓ} might not be a decreasing function of the price $p(\mathbf{z})$. That would only hold however with high values of β.

Frictionless equilibrium The demand for housing at each location $\mathbf{z} \in \mathcal{Z}$ is equal to the supply of units.

$$D(\mathbf{z}; f()) = S(\mathbf{z}),$$

which in turn implies:

$$D(\mathbf{z}; f()) = h(\mathbf{z}) - D(\mathbf{z}; f())$$

and thus:

$$D(\mathbf{z}; f()) = \frac{h(\mathbf{z})}{2}$$

which implies that price differences reflect (i) amenity differences and (ii) the supply of housing in the neighborhood. This is formalized as follows.

Definition. An equilibrium price schedule is a function $p : \mathbf{z} \mapsto p(\mathbf{z})$ such that the demand for housing in each location is equal to half the number of housing units in that location.

Hence an increase in the number of housing units $h(\mathbf{z})$ will lead to a decline in the equilibrium price $p(\mathbf{z})$, at constant amenities.

Remark. This relationship implies a constraint on the number of occupied and vacant units: as $\int D = N$, and $\int h = H$, we need $N = H/2$. In other words, it is the arrival rate of offers and the uncertainty of matches that implies the imbalance between N and H in the frictional model.

The price amenity gradient With a one-dimensional amenity, and a constant supply of housing across the $\mathbf{z}$, the price-amenity gradient exactly reflects households' preferences for amenities:

$$(\gamma - \frac{dp}{dz}) \cdot D(1 - D) = 0 \Rightarrow \frac{dp}{dz} = \gamma \tag{39}$$

With a varying supply, the price-amenity gradient reflects both preferences γ and the gradient of supply w.r.t. amenities:

$$(\gamma - \frac{dp}{dz}) \cdot D(1 - D) = \frac{1}{2}\frac{dh}{dz} \Rightarrow \frac{dp}{dz} = \gamma - \frac{1}{2D(1 - D)}\frac{dh}{dz} \tag{40}$$